Online Appendix

Appendix A: The log-t convergence test

PS consider the following nonlinear dynamic factor model for y_{it} of the form

$$\log y_{it} = a_{it} + x_{it}t = \left(\frac{a_{it} + x_{it}t}{\mu_t}\right)\mu_t = b_{it}\mu_t,$$

where the component a_{it} embodies transitional dynamics for real per capita income and the component $x_{it}t$ captures the idiosyncratic time paths of technological progress. Note that both components are allowed to be heterogeneous across individuals and over time. The dynamic factor formulation $b_{it}\mu_t$ involves a common growth component, μ_t, which may represent a proxy for commonly available technology, and individual transition factors (b_{it}) that measure the transition path of an individual economy to μ_t. The transition elements b_{it} is further modeled as

$$h_{it} = \frac{\log y_{it}}{N^{-1}\sum_{i=1}^{N}\log y_{it}} = \frac{b_{it}}{N^{-1}\sum_{i=1}^{N} b_{it}},$$

where h_{it} is called the *relative transition path* as it measures economy i's relative departure from μ_t. If there is a common transition across economies, $h_{it} = h_t$ and $h_{it} \to 1$ for all i as $t \to \infty$. This implies that relative convergence exists if

$$\lim_{t\to\infty}\frac{\log y_{it}}{\log y_{jt}} = 1, \quad \text{for all} i \text{ and } j.$$

Also, under convergence, the mean square transition differential is

$$H_t = \frac{\sum_{i=1}^{N}(h_{it}-1)^2}{N} \to 0 \quad \text{as } t \to \infty.$$

PS use a semiparametric model for the transition coefficients allowing for heterogeneity across individuals and over time as

$$b_{it} = b_i + \frac{\sigma_i \zeta_{it}}{L(t)t^{\alpha}},$$

where b_i is fixed, ζ_{it} is $iid(0,1)$ across i but may be weakly dependent over t, and $L(t)$ is a slowly varying function (like log t) for which $L(t) \to \infty$ as $t \to \infty$. Note that the parameter α governs the rate at which the cross-section variation over the transitions decays to zero over time.

Then, define H_t as the quadratic distance measure of

$$H_t = N^{-1}\sum_{i=1}^{N}(h_{it}-1)^2,$$

where h_{it} denotes the *relative transition coefficient* which is obtained either from a prefiltering method or directly from the data in

$$h_{it} = \frac{\log y_{it}}{N^{-1}\sum_{i=1}^{N}\log y_{it}} = \frac{b_{it}}{N^{-1}\sum_{i=1}^{N} b_{it}}$$

which eliminates a common component by scaling and measures the transition element for economy i relative to the cross-section average.

The null hypothesis of convergence can then be formulated as

$$H_0 : b_i = b \;\text{ and }\; \alpha \geq 0,$$

against the alternative hypothesis of

$$H_A : \{b_i = b \;\text{ for all } i \;\text{ with } \alpha < 0 \;\} \;\text{ or }\; \{b_i \neq b \;\text{ for some } i \;\text{ with } \alpha \geq 0, \;\text{ or } \alpha < 0\}.$$

Under the null, the transition distance H_t has the limiting form of

$$H_t \sim \frac{A}{L(t)^2 t^{2\alpha}} \quad \text{as} \quad t \to \infty.$$

By setting $L(t) = \log t$, we obtain the following regression model

$$\log \frac{H_1}{H_t} - 2\log(\log)t = a + \gamma \log t + u_t, \quad \text{for} t = T_0, ..., T.$$

The initial observation in the regression is $T_0 = [rT]$ for some $r > 0$ so that the first r% of the data is discarded. $r \in [0.2, 0.3]$ is suggested by Phillips and Sul (2007). Under the null of convergence, the point estimate of the parameter γ converges in probability to the scaled speed of convergence parameter 2α. The corresponding t-statistic in the regression is constructed in the usual way using HAC standard errors. This t-statistic diverges to positive infinity when $\alpha > 0$ and converges weakly to a standard normal distribution when $\alpha = 0$. The convergence test then proceeds as a one-sided t-test of $\alpha \geq 0$. Under the alternative of divergence or club convergence, the point estimate of γ converges to zero regardless of the true value of α, but its t-statistic diverges to negative infinity, thereby giving the one-sided t-test discriminatory power against these alternatives. Since the log-t regression test has power against club convergence alternatives, the procedure can also facilitate the discovery of club convergence clusters.

Appendix B: Clustering mechanism

The *clustering mechanism procedure* involves the following stepwise and cross section recursive application of log-t regression tests. The reader is referred to their work for further details.

Step 1 (Cross-section ordering): Order the entire series according to either the amount of final period income or the average of the last half period of incomes.

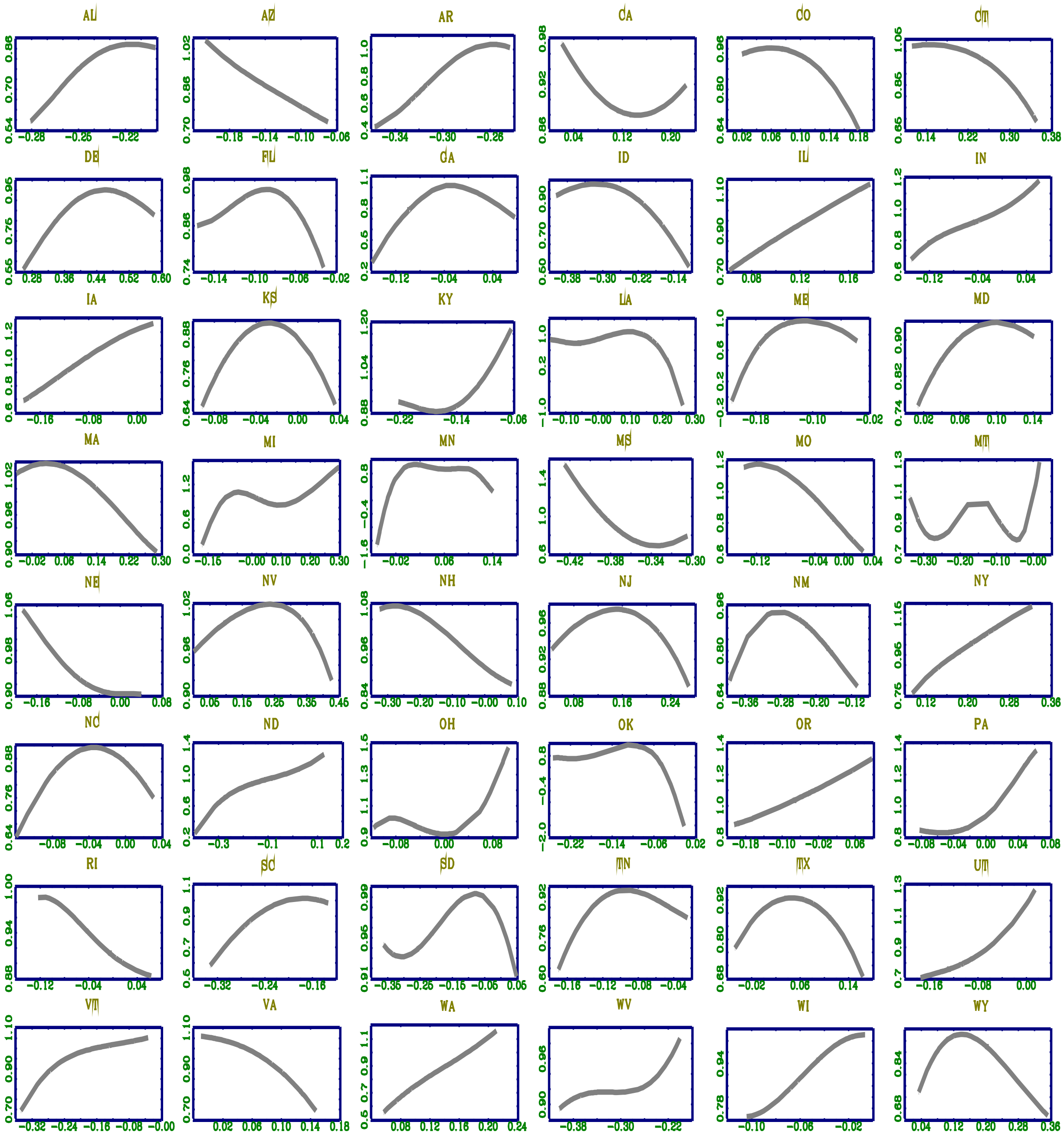
Step 2 (Form a core primary group): Select the first k individual series in the panel to form a subgroup G_k for some $2 \leq k < N$. Then run the log-t regression and calculate the convergence test statistic $t_k = t(G_k)$ for this subgroup. Choose the core group size k^* by maximizing t_k over k according to the criterion

$$k^* = \text{argmax}_k\{t_k\} \quad \text{subject to } \min\{t_k\} > -1.65.$$

If the condition min $\{t_k\} > -1.65$ does not hold for $k = 2$, then the highest individual in G_k can be dropped from each subgroup and new subgroups $G_{2j} = \{2, ..., j\}$ formed for $3 \leq j \leq N$. The step can be repeated with test statistics $t_j = t(G_{2j})$.

Step 3 (Sieve the data for new club members): Add one series at a time to the core primary group with k^* members and run the log$t$ test again. Include the new series in the convergence club if the associated $t-$statistic is greater than the criterion c^*.

Step 4 (Recursion and stopping rule): Form a second group from those series for which the sieve condition fails in Step 3. Run the log-t test to see if $t_{\hat{\gamma}} > 1.65$ on this group, i.e., if this group satisfies the convergence test. If so, conclude that there are two convergence club groups: the core primary group and the second group. If not, repeat steps 1 through 3 to see if this second group can itself be subdivided into convergence clusters. If there is no k in Step 2 for which $t_k > -1.65$, conclude that the remaining series do not contain a convergence subgroup and thus the remaining states have divergent behavior.



A complete version of Figure 4