Inequalities: Fun With Loops

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GK-12 MAVS Mentor Teacher – Kimberly Helixon

Class: Algebra 1

9th grade – 1st six weeks

Topic: Inequalities

Objectives: Students will gain experience with inequalities. Students will use problem solving strategy to force other players out of the game we will play. Students will relate inequalities to graphs.

Standards: TEKS –	(7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to: (B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and (C) interpret and determine the reasonableness of solutions to linear equations and inequalities.
NCTM 2000 Algebra	
Represent and analyze mathematical situations and structures using algebraic symbols	• understand the meaning of equivalent forms of expressions, equations, inequalities, and relations; • write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases; • use symbolic algebra to represent and explain mathematical relationships;
Key Vocabulary:	Inequality, less than, greater than
Materials and Resources:	Powerpoint to run the activity, followup materials from Engaging Mathematics Algebra 1, pgs. 211 and 213
Research Setting / Connection / Motivation	If…while… loops are used in programming and incorporate inequalities to perpetuate the loop.
Prior Knowledge:	Students are supposed to know the less than symbol and the greater than symbol, but often they do not.

Lesson Presentation:

Engage: Iván will start with the powerpoint that explains who he is and what he is researching. Then he will set up a game for the students. The class will play the game as a whole. The goal is to stay "in" the game and not get "out". The condition to start the game is: $x > 6$. As long as your number is greater than 6, you stay "in" the game. If your number ends up being 6 or less, then you are "out". There is strategy involved because students can force the next player to be "out" if they plan appropriately. We will play several rounds. Each time we will change the conditions of the game as follows:

Round 1: $x > 6$, we will start with the number 20, students will be allowed to subtract 1, 2, or 3

Round 2: $x \leq -3$, we will start with the number -50, students will be allowed to add 1, 2, 3, 4, or 5

Round 3: $x \geq -20$, we will start with 145, students will be allowed to subtract 5 or 10

Explain: Students who "lose" will then get the chance to play the computer. Iván will set up a simple program loop that will allow the kids to come up and input numbers to compete with the computer. When the student or computer "fails", a screen will pop up that says they are "out". This is a very simple way to show what happens with computer games.

Explore: Students will form groups of two. Each group is one given page 211 from Engaging Mathematics Algebra 1. Students will work their way through the tables given and circle True or False based on the problem given. Then based on their answers, they will graph their solution below on the provided number line. The two tables and graphs are simple because there are only positives. Once all the groups are done with pg. 211, they will then be given pg. 213. These are harder because there are negatives.

The idea is to have the students generate the rule that when you divide by a negative, the inequality flips. This is taught in the previous year, but many students don't remember and many don't know why it works.

Questions to help them get there:
What do you notice about the arrows on the first problem on each page?
Why did they go in opposite directions?
What about the second problem on each page?
Can you generate a rule that could explain why this happened?

Extend: Students will then be shown a series of inequalities and asked whether or not the inequality sign will flip, assuming they solve it by dividing. They are not solving the problems, just stating whether it will flip. Students will be asked to tell "why".

Explore: Students will then be shown a series of inequalities and asked which way the graph is going to go. All the students will stand up at their seats and raise their right arm for graphs going right, and raise their left arm for graphs going left. There will be negatives that cause the inequality symbol to

flip and therefore the graph will go the other way. I will keep all the variables on the left to begin.

Extend: Students will then be given problems with the variable on the right hand side and repeat the activity where they stand and raise their right arm or left arm depending on which way the graph should go. I will also have negatives to try and trip them up.

Closing/Evaluation: The evaluation is built in to the activity. I will know immediately who is getting it and who isn't based on the arm activity. Iván will wrap up by going back and summarizing If…While… loops.

Vertical Strand: Algebra – Investigating Methods for Formulating and Solving Equations

6.4A use tables and symbols to represent and describe proportional and other relationships such as those involving conversions, arithmetic sequences (with a constant rate of change), perimeter and area, *such as input/output*

7.5A use concrete and pictorial models to solve equations and use symbols to record the actions

A.7B investigate methods for methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities

A.7C interpret and determine the reasonableness of solutions to linear equations and inequalities

2A.3C interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts;

2A.6A determine the reasonable domain and range values of quadratic functions, as well as interpret and determine the reasonableness of solutions to quadratic equations and inequalities;

Teacher Notes:

Student Worksheets / Handouts / Powerpoint	Engaging Mathematics Algebra 1, pgs. 211 and 213, powerpoint to run the activity, computer program using simple If…While… logic.

Student Name: _____ Date: _____

True or Not?

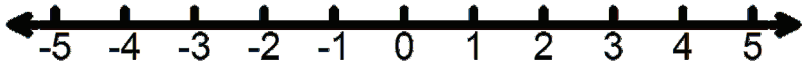
Substitute each value in the table into the inequality at the top. If the resulting inequality is true, shade the box with T. If resulting inequality is false, shade the box with F.

$5m \leq 10$		
$m = -5$	T	F
$m = -4$	T	F
$m = -3$	T	F
$m = -2$	T	F
$m = -1$	T	F
$m = 0$	T	F
$m = 1$	T	F
$m = 2$	T	F
$m = 3$	T	F
$m = 4$	T	F
$m = 5$	T	F

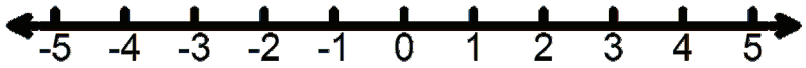
$a + 2 > 0$		
$a = -5$	T	F
$a = -4$	T	F
$a = -3$	T	F
$a = -2$	T	F
$a = -1$	T	F
$a = 0$	T	F
$a = 1$	T	F
$a = 2$	T	F
$a = 3$	T	F
$a = 4$	T	F
$a = 5$	T	F

On each number line below, place dots on the values that made the inequality true.

$5m \leq 10$



$a + 2 > 0$



Communicating About Mathematics

In the first table, how would values of m between 2 and 3 be shaded? Why?

_____

_____

_____

_____

_____

Student Name: _____ Date: _____

True or False?

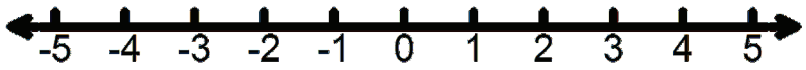
Substitute each value in the table into the inequality at the top. If the resulting inequality is true, shade the box with T. If resulting inequality is false, shade the box with F.

$-3x \leq 9$		
$x = -5$	T	F
$x = -4$	T	F
$x = -3$	T	F
$x = -2$	T	F
$x = -1$	T	F
$x = 0$	T	F
$x = 1$	T	F
$x = 2$	T	F
$x = 3$	T	F
$x = 4$	T	F
$x = 5$	T	F

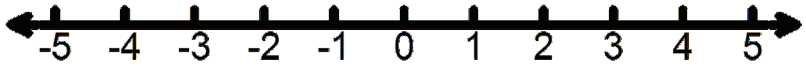
$-2y > -6$		
$y = -5$	T	F
$y = -4$	T	F
$y = -3$	T	F
$y = -2$	T	F
$y = -1$	T	F
$y = 0$	T	F
$y = 1$	T	F
$y = 2$	T	F
$y = 3$	T	F
$y = 4$	T	F
$y = 5$	T	F

Transfer your answer to the number line below by placing dots on the values that made the inequality true.

$-3x \leq 9$



$-2y > -6$



Communicating About Mathematics
Write a sentence describing the values that make each inequality true.

_____

_____

_____

_____

_____

Student Name: Key Date: _____

True or Not?

Substitute each value in the table into the inequality at the top. If the resulting inequality is true, shade the box with T. If resulting inequality is false, shade the box with F.

$5m \leq 10$

$m \leq 2$

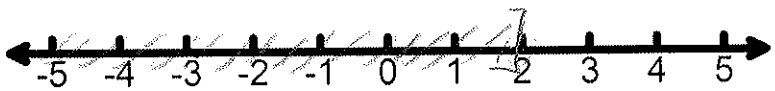
$5m \leq 10$		
$m = -5$	T	F
$m = -4$	T	F
$m = -3$	T	F
$m = -2$	T	F
$m = -1$	T	F
$m = 0$	T	F
$m = 1$	T	F
$m = 2$	T	F
$m = 3$	T	F
$m = 4$	T	F
$m = 5$	T	F

$a > -2$

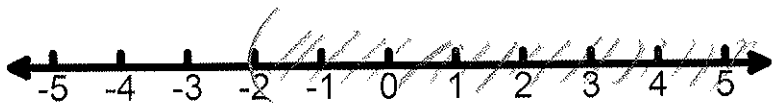
$a + 2 > 0$		
$a = -5$	T	F
$a = -4$	T	F
$a = -3$	T	F
$a = -2$	T	F
$a = -1$	T	F
$a = 0$	T	F
$a = 1$	T	F
$a = 2$	T	F
$a = 3$	T	F
$a = 4$	T	F
$a = 5$	T	F

On each number line below, place dots on the values that made the inequality true.

$5m \leq 10$

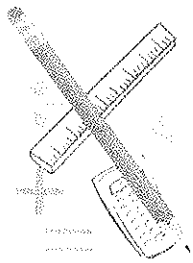


$a + 2 > 0$



Communicating About Mathematics

In the first table, how would values of m between 2 and 3 be shaded? Why?



Student Name: Key Date: _____

True or False?

Substitute each value in the table into the inequality at the top. If the resulting inequality is true, shade the box with T. If resulting inequality is false, shade the box with F.

$-3x \leq 9$

$\frac{-3x}{-3} \leq \frac{9}{-3}$

$x \geq -3$

$-3x \leq 9$		
$x = -5$	T	(F)
$x = -4$	T	(F)
$x = -3$	(T)	F
$x = -2$	(T)	F
$x = -1$	(T)	F
$x = 0$	(T)	F
$x = 1$	(T)	F
$x = 2$	(T)	F
$x = 3$	(T)	F
$x = 4$	(T)	F
$x = 5$	(T)	F

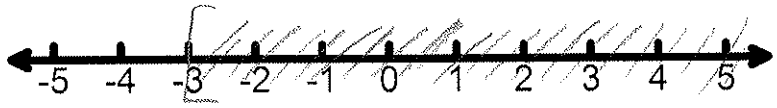
$-2y > -6$

$y < 3$

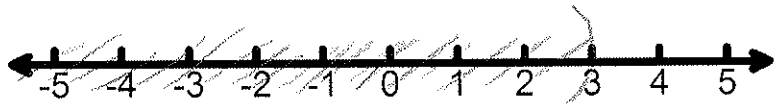
$-2y > -6$		
$y = -5$	(T)	F
$y = -4$	(T)	F
$y = -3$	(T)	F
$y = -2$	(T)	F
$y = -1$	(T)	F
$y = 0$	(T)	F
$y = 1$	(T)	F
$y = 2$	(T)	F
$y = 3$	T	(F)
$y = 4$	T	(F)
$y = 5$	T	(F)

Transfer your answer to the number line below by placing dots on the values that made the inequality true.

$-3x \leq 9$

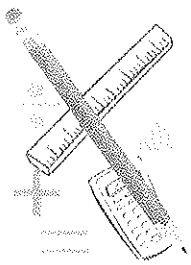


$-2y > -6$



Communicating About Mathematics

Write a sentence describing the values that make each inequality true.



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Inequalities: Fun With Loops

Iván Ojeda

What is this symbol?

- <
- Less than
- >
- Greater than
- ≤
- Less than or equal to
- ≥
- Greater than or equal to

How do you read these?

- $7 < 20$
- $50 > 22$
- $x > 10$ Give me some examples
- $x < 15$ Give me some examples
- $x \geq -20$ Give me some examples
- $2 \leq x$ (read from the x, then give an examples)

- $10 \geq x$ (read from the x, then give an example)

Mr. Ojeda's research

- If…

  x > 50 the program continues

- While…

  once x is NOT > 50, then it ends

Let's play a game

- We will divide the class in half
- Half the group will get a number to wear around your neck
- The other half will be the audience

Now let's see what you know:

- Pair up with a friend
- Each of you needs pg. 211
- Circle whether the statements are True or False in each of the tables
- Place a dot on the values that were True on the number lines below
- We will discuss it when you are done

Now let's make it harder:

- Turn your paper over to pg. 213
- Circle whether the statements are True or False in each of the tables (be careful since there are negatives!)
- Place a dot on the values that were True on the number lines below
- We will discuss it when you are done

What did you notice?

- What do you notice about the arrows on the first problem on each page?
- Why did they go in opposite directions?
- What about the second problem on each page?
- Can you generate a rule that could explain why this happened?

Solving Inequalities

- You can solve inequalities to find the solution.
- Example:

$$\frac{5m}{5} \leq \frac{10}{5}$$

$$m \leq 2$$

Solving Inequalities

- When you divide by a negative number the inequality is flipped
- Example:

$$\frac{-3x}{-3} \leq \frac{9}{-3}$$

$$x \geq -3$$

Assuming you divide, which of these will flip?

- $3x \leq -21$
- $-4x \geq -80$
- $3x > 12$
- $-5x < 30$
- $10x \leq -50$

Wrap Up:

- If… While… logic is used to keep loops going in programming. Inequalities are used for this.
- Do you always want loops to continue forever?
- When would you want a loop to continue?
- When would you want a loop to end?

Lesson Reflection:

In this lesson we worked on the idea of inequality with the students. When introducing variables to inequalities there's a barrier that students have to break in order to understand the concept. That's why using several examples in the beginning of the class helps them get engaged. The idea of an arbitrary value is what gives us the power to make a decision. An inequality represents the numerical value the computer expects and makes a decision based on that value. The computer does this several times until it reaches the desired value. This is an essential part of my research and we try to make it as clear as possible at the beginning of the lesson by giving other examples (ex: get paid until you have enough money to buy something, run until you reach the desired amount of miles, etc.)

On the second part of the game we have the students hang a number around their neck and the other half of the class has to tell them if they fit in the inequality or not. This exercise is great to get them moving and being more active.

The third part of the lesson consists of letting them figure out which values satisfy the inequality. With those values they can create a graph that helps them understand the depth of the concept. This worksheet contain the concept of "flipping inequalities" when dividing by a negative number and they can discover it by themselves