## NSF GK-12 MAVS Project Lesson Plan

GK-12 MAVS Fellow: Larrissa Owens
GK-12 MAVS Mentor Teacher: Daree Yancey
Lesson Title: Things That Affect Rates
Class: $7^{\text {th }}$ Grade Math Pre-Algebra
Topic: Rates
Objectives: Students will solve for several unit rates but also gain insight into the fact that rates are not always constant. They will dive into a game modeling a predator-prey model and explore of the factors that influence the rate of cell populations, and will solve simplified problems requiring application of unit rate and death rate knowledge. Students will also explore examples and non-examples of key vocabulary related to rates.

Standards TEKS-(8.1)(B )To select and use appropriate forms of rational numbers, (8.2)(D) To use multiplication by a constant factor (unit rate), (8.3)(B) To estimate and find solutions to application problems involving rates, (8.5)(B) find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change) Standards NCTM- 1. Number and Operations, 4. Measurement, 6. Problem Solving, 7. Reasoning and Proof, 8. Communication, 9. Connections, 10. Representation

Key vocabulary: Debris, Macrophage, Apoptosis, Rates, Unit Rates, Death Rates, Scale Factor, Similar Figures, Proportion, Ratio

Materials and Resources: Checker Board, PacMan Tiles, PacMan Checkers rules sheet, Powerpoint/smart-board, vocabulary boxes sheet, warm-up "score sheet", homework "Rates"

Time Required: 45-60 min
Research Setting/Connection/Motivation: When using mathematical systems to model the inflammatory response the cell populations are the root of the reaction terms that are affecting and driving the healing process. These cell populations are modeled mathematically by including death rates, growth rates and the interaction effects of cells. The effects on these rates are crucial in modeling these populations since they rarely can be modeled with a strictly linear growth or decay rate. The students will be able to see that both the population of macrophages and the population of debris present effect the rates at which each of those populations are growing and decaying.

Prior knowledge: Students would need to be able to work with ratios and division. Students are assumed to have already learned the term carrying capacity.

Vertical Strands: TEKS-(8.1)(B )To select and use appropriate forms of rational numbers, (8.2)(D) To use multiplication by a constant factor (unit rate), (8.3)(B) To estimate and find solutions to application problems involving rates, (8.5)(B) find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change)College Algebra (slope and interpretations of steepness), Pre-Calculus (Non-linear functions), Calculus(Slope fields and interpreting slope as a rate of change), ODE (modeling rates of populations, observing transient behavior)


## Math Journal

Friday: Explain ...(question related to previous day's lesson)

| Score Sheet |  |  |
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| 6 7 8 |  | - recoro am |
| $\stackrel{9}{10}$ |  | manio |
|  |  | every |

## Larrissa Owens

 Graduate Students, GK12 Fellow Department of Mathematics, UTA

SIGHTS SET ON ADVANCING TREATMENT FOR RETINAL DISEASE


- What is the carrying capacity of the PacMan population?

- What is the carrying capacity of the PacMan population?
 the game is 6


## A closer look at Macrophage growth rates



## (Recall) When Macrophages reach their carrying capacity it will stop multiplying

$$
\begin{aligned}
& \text { Notice from day } 15 \text { till day } 30 \text { we do } \\
& \text { not see any notable gain or lose of } \\
& \text { cells, so notice that the growth rate is } \\
& \text { constant and equal to } 0 \text { over this time } \\
& \text { interval: }
\end{aligned}
$$

Growth Rate $=\frac{\text { (Cells at } 30 \text { days)-(cells at } 15 \text { days) }}{30-15}$

$$
=\frac{1-1}{30-15}=0
$$

- What would happen to the ghosts (Debri) if there were no PacMen (Macrophages)?

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$$
\frac{d D}{d t}=-f_{0} \lambda_{1} M D+f_{0} \lambda_{3} M
$$



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$$
\frac{d D}{d t}=-f_{0} \lambda_{1} M D+f_{0} \lambda_{3} M
$$

*Debri population is not changing, the rate of change is 0 , we will maintain the same amount that
 we started with

- What would happen to the ghosts (Debri) if there were no PacMen (Macrophages)?

$$
\frac{\partial d}{\partial t}=D_{0} \nabla^{2} d-f_{0} M_{1} d+\tilde{f}_{0} M_{3}
$$

*There are ways to account for cell movement in the math modeling.


- Since PacMan represents a Macrophage cell, what do you think it means for PacMan to lose a life?

$$
\frac{d M}{d t}=a_{11} M C H\left(M_{0}-M\right)-a_{0} M
$$

Apoptosis: Programed cell death

$$
\frac{d M}{d t}=a_{11} M C H\left(M_{0}-M\right)-a_{0} M
$$

- Ghosts, did anyone die in the first round? The second?

When we have a high balance of macrophages and debris cells then the debris will be dying faster


Debris are "safe" for at least 2 turns

When we have a high balance of macrophages and debris cells then the debris will be dying faster



## Things that affect rates: (cells)/(time)

- Carrying capacity of a population
- The amount of predators and prey that are present
- The apoptosis rate


## Vocabulary Boxes- All About Proportions



## Vocabulary Boxes- All About Proportions



| Rate | - <br> (Assuming a constant rate) If 84 bacteria |
| :---: | :---: |
| Definition: <br> A Rate is a ratio that compares quantities measured in different units. | what is the death rate |
|  | Noreample: |
| vorom Omefintion: |  |
| $\mathfrak{H} \mathbb{N}$ | How is it different BiN |


| Rate | Example: <br> (Assuming a constant rate) If 84 bacteria |
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| Definition: <br> A Rate is a ratio that compares quantities measured in different units. | are killed over the course of 3 hours what is the death rate. $\frac{\text { bacteria }}{\text { hour }}=\frac{84}{3}$ |
|  | Non-example: |
| Your Own Definition: |  |
|  | How is it different from a ratio? Fivo |


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|  | The percent of change of your grade from last six weeks to this six weeks |
| Your Own Definition: | $\frac{\text { New grade-Old grade }}{\text { Old grade }} \times 100$ |
|  | How is it different from a ratio? Fion |


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| Your Own Definition: | $\frac{\text { New grate-Old grade }}{\text { Old grade }} \times 100$ <br> Mnics |
|  | How is it different from a ratio? $5 \sqrt[5]{ }$ |



| Unit Rate | (Assumpmple. a constant rate) If 84 bacteria are killed over the course of 3 hours what is the death rate per |
| :---: | :---: |
| Definition: <br> A Unit Rate is a rate with a denominator of 1 | hour. $\frac{\text { bacteria }}{\text { hour }}=\frac{84}{3}=\frac{28}{1}$ |
|  | You buy a 51 b bag of Oranges for $\$ 3$. What is the cost per pound. <br> lbs 5 |
| Your Own Definitio | \$ 3 |
|  | Howto |


| Unit Rate | Smame: |
| :---: | :---: |
| Definition: <br> A Unit Rate is a rate with a denominator of 1 | $\frac{\text { bacteria }}{\text { hour }}=\frac{84}{3}=\frac{28}{1}$ |
|  | Non-example: <br> You buy a 5lb bag of Oranges for $\$ 3$. What is the cost per pound. |
| $\mathrm{BON}$ | Set up the desired rate and simplify (or divide) until the denominator is 1 |



Similar Figures

| Scale Factor |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| Definition: <br> Scale Factor is the ratio <br> that relates the scaled <br> measurement to the <br> actual measurement |  |  |  |  |  |  |  |




| Scale Factor |  |  |
| :---: | :---: | :---: |
| Definition: <br> Scale Factor is the ratio that relates the scaled measurement to the actual measurement | $\xrightarrow{300}$ |  |
|  | Nonemomple: |  |
| Vorormomefinton: | $1200 \% 1$ |  |
|  | How to find it: <br> SF = | $N$ (New) |
|  |  | O (Original) |

Name


RATES


1. There are 5 ghosts and you have 7 turns to kill them. How many ghosts must you kill per turn?

## Workspace

How do you know your answer is correct?
2. You are killing ghost at the rate of $2 / 3$ ghosts each turn. How many turns will it take to kill 6 ghosts.

Workspace

How do you know your answer is correct?
3. You kill 4 bacteria over the course of 6 hours. What is your victory rate in terms of bacteria killed per hour?

[^0]4 Lori wants to go to the beach on Saturday but isn't sure if she has enough gas in her car to drive there and back home. The beach is 77 miles from Lori's house, and her car gets about 28 miles oer

## Lesson Presentation Lesson 4:

- Engage: Students will verbalize their learning from the previous day in their math journal and then play Pacman and Ghost Checkers to introduce a visual representation of a predator-prey model.
- Explore: Class discusses several things they observed in the checkers game and see how each component is modeled in research equations and graphs
- Explain: Students see examples/videos of what things that affect rates at the cellular level and learn what apoptosis is
- Extend: Students begin to see that rates are not always constant but can see how even rates that are constant are a very powerful tool. Students are led through several examples and non-examples to review key terms related to rates.
- Evaluate: Students will self-evaluate when explaining their reasonableness to the responses on their worksheets and when they are asked to rewrite key definitions in their own words.


## PacMan Checkers Rules

Board Set: you need 5 ghosts on white squares nearest that player, and 2 PacMan on the white squares of opposite end (The remaining 4 pacmen pieces will stay off the board until you are instructed to gain a life). Record these numbers on the Score Sheet next to "Round 0".

## Round 1:

- PacMan moves
- Ghost Moves
- PacMan gains a life
- Record data


## Round 2:

- PacMan moves
- Ghost Moves
- PacMan loses a life
- PacMan gains a life
- Record data


## Round 3

- PacMan moves twice in a row
- Ghost Moves
- PacMan gains a life


Move: A diagonal move in the forward direction on to an open white square or a diagonal jump over an enemy (this results in that enemy being removed from the board)

Gain a life: If there remains a PacMan not in use place it anywhere in the two rows closest to you.

Lose a life: Remove a PacMan of your choice from the game board.

- Record data

Repeat until only one player's pieces remain.

## Notes:

- You can only use the pieces provided (5 ghosts, 6 PacMen)
- No double jumps
- Gaining a life does not count as your turn
- Once you reach the opposite side of the game board you can begin moving in a backward direction
- If your last piece has no open moves it dies



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B & B & B & B & B & B \\
B & B & B & B & B & B
\end{array}
$$

$00000000000$

Names $\qquad$
Score Sheet
PacMan Level 3

| Round | PacMan | Ghost |
| :---: | :--- | :--- |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
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| 13 |  |  |
| 14 |  |  |
| 15 |  |  |
| 16 |  |  |
| 17 |  |  |
| 18 |  |  |
| 19 |  |  |
| 20 |  |  |

## Reflections Lesson 4:

The checkers engagement led to really good observations about "fairness" as the students realized that several factors play into who "wins" in a predator-prey model. Students enjoyed getting to see what looks like when a cell undergoes apoptosis and it led to several more questions about what the mathematics is able to capture and how it relates to the real biology. Students seemed to pay attention through the discussion about things that affect rate, but especially later in the day the need for me to keep asking them questions so that they could connect to and own the lesson was critical to keeping them engaged. The vocabulary review was helpful from the curriculum standpoint because the students were gearing up to take a Curriculum Assessment, but was also really cool because I could reference things we had discussed in a previous lesson. This was my first time using Vocabulary Boxes but now after having seen the impact that they can have in clarifying closely related terms for the students I hope to incorporate them more into future lesson planning.

## Teacher Notes Lesson 4:

This lesson fit well in the 45-60 min class period range. Students will get different results from the checker activity but some of the things that you will want to discuss with them is the fact that in most cases the pacmen won even though it started out with fewer pieces looking at the design of the rules that accounted for this and the ways the death rates relate to biology really help the students to understand the goal of the activity. During the scale factor review, emphasizing that when the scaled model is smaller than the original, un-scaled object then the scale factor is less than one was a beneficial point to continue instilling in the students mind. Laminating (or putting in sleeves) the checkers boards and rules helped keep them from walking off over the course of the day so that we only had to run 15 copies.

| Proportion | Example: | In order to heal we need the debri population to be 20\% of the Macrophage population. If we start with 2 debri how many Macrophages do we |
| :---: | :---: | :---: |
| Definition: <br> A Proportion is an equality of two ratios. |  | $\frac{2}{x}=\frac{20}{100} \quad x=10$ |
|  | Non-example: |  |
| Your Own Definition: | How is it different from a ratio? <br> It takes 2 ratios, set equal to each other |  |


| Similar Figures | Example: |
| :---: | :---: |
| Definition: <br> Similar Figures are figure in which all corresponding sides have proportional lengths | $3 \quad 6$ |
|  | Non-example: |
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$\qquad$ KEY $\qquad$

## RATES

| 1. There are 15 ghosts and you have 3 turns to kill them. How many ghosts must you kill per turn? <br> Workspace | 2. You are killing ghost at the rate of $2 / 3$ ghosts each turn. How many turns will it take to kill 6 ghosts. <br> Workspace |
| :---: | :---: |
| Ghosts 15 | 6 2 |
| $\overline{\text { turns }}=\varlimsup_{3}=5 \text { ghosts per turn }$ | $\overline{x \text { (turns) }}=\frac{}{3}=(6 \times 3) / 2=9 \text { turns }$ |
| How do you know your answer is correct? <br> If I kill 5 ghosts each turn for 3 turns I multiply those and get 15 |  |
|  | How do you know your answer is correct? If I multiply 9 times (2/3) I get 6 . |
| 3. You kill 4 ghosts over the course of 6 turns. What is your victory rate in terms of ghosts killed per turn (assume you can kill a fraction of a ghost)? <br> Workspace | 4 Lori wants to go to the beach on Saturday but isn't sure if she has enough gas in her car to drive there and back home. The beach is 77 miles from Lori's house, and her car gets about 28 miles per gallon. How many gallons of gas does Lori need to have in her car to make the trip to the beach? Workspace |
| $\underline{\text { Ghosts }}={ }^{4}=. \overline{66} \text { ghosts per turn }$ | miles $77$ |
| turns 6 | $\overline{\text { gallon }}=28=\frac{}{x}, x=77 / 28=2.75 \text { gal. }$ |
| How do you know your answer is correct? If I multiply 6 times (.66) I get 4. | But that is only one way, so we multiply this by 2 and get: 5.5 gal. |
|  | How do you know your answer is correct? |
|  | If I multiply 2.75 times 28 I get the 77 miles I need. |

Name $\qquad$ RATES

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Workspace

How do you know your answer is correct?
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How do you know your answer is correct?


[^0]:    Workspace

