Optimizing Equations

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Class: Algebra 1 | ***9^{th} grade – 1^{st} six weeks***

Topic: Why we solve equations in a certain order and what is that order

Objectives: Students will experience/discover why the steps of an equations are important. Students will apply the steps for solving equations to a series of equations requiring combing like terms, x on both sides, and the use of reciprocals.

Standards: TEKS –	(7) Linear functions. The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to: (B) investigate methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities; and (C) interpret and determine the reasonableness of solutions to linear equations and inequalities.
NCTM Algebra 2000	
Represent and analyze mathematical situations and structures using algebraic symbols	• understand the meaning of equivalent forms of expressions, equations, inequalities, and relations; • write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases; • use symbolic algebra to represent and explain mathematical relationships
Key Vocabulary:	Equation, inverse operation, reciprocal, optimize
Materials and Resources:	Five equations to be used with all 5 of the groups, chart to keep track of each groups' number of steps, practice problems to use after the game, powerpoint to use after the game and use the promethean board to show the steps
Research Setting / Connection / Motivation	My research is all about finding the fastest way to do things.
Prior Knowledge:	Students should know that what you do to one side of an equation, you do to the other side. Students should have an understanding of integers.

Name:________________________ Period:___________

Solve the following. Use the nsolve button.

1) $3x - 4 = 20$

2) $2x + 24 = 6x - 8$

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Lesson Presentation:

Engage: Iván will explain what he does again so the students will remember. He will explain that the purpose of this lesson is to help the students become more efficient in solving equations. We will play a game to do this. Students will be divided into groups of four or five, creating a total of five teams. Each team will have some sort of icon to use to keep track of their total number of steps during the game.

Explore: Each team will be given an equation to solve. The equation for each group will be different so that the teams cannot simply copy another team's work. Each team will send one member to the chalk board to write down the equation and perform the first step of solving. Either Iván or I will then make sure that the team writes the result of that step down correctly and moves their icon 2 spaces forward to represent the 2 steps that have been performed (one on each side). The team then tries to do the next step. Again, either Iván or I will confirm that the next step is written down correctly and the icon is moved 2 steps forward. At any time during the game, a team can choose to start over or erase a step, but the icon never gets to move backwards. In essence, they will suffer a penalty if they do not proceed through the equation in an efficient way. The goal of the game is to have the least number of steps at the end. Each group will get the chance to 5 equations and each group will do the same five equations. However, since cheating can be a problem, they will get them in a different order. The five equations will be:

$5x + 3 = 3x + 9$ answer is 3

$\frac{3x}{4} + 7 = 10$ answer is 4

$2x - 6 = 20$ answer is 13

$7x + 1 - 2x + 8 = 24$ answer is 3

$39 = 11x - 5$ answer is 4

Notice that the equations do not have too many weird negatives to have to deal with. That is deliberate since the focus is building the concept of solving equations and why the steps matter.

Explain: At the end of the game, we will go back over each problem and determine the least number of steps to solve it and WHY we do the steps in that specific order. Additional problems of the same type will be given to the teams to practice before moving on to the next equation. The steps for each problem will be recorded to see if there are any patterns that we can discover and use. The practice will take place on white boards and kids can hold them up to show they have it done.

Extend: The extension will be to give them a problem completely different from the previous kind and see if a group and figure out the most efficient way to solve it. A good problem for this might be $\frac{5x-1}{2} = 12$. The answer is 5. This problem does not follow the pattern exactly due to the denominator.

Closing/Extension: Iván will wrap up by showing students a summation notation problem that shows how a computer optimizes summing numbers.

$$\sum_{n=0}^{30} 2^n$$

$$= 2^0 + 2^1 + 2^2 + 2^3 + \ldots. + 2^{30}$$

SLOW!!! because of the repeated multiplication

$$= 1 + 2(1 + 2^1 + 2^2 + \ldots + 2^{29})$$

$$= 1 + 2(1 + 2(+ 1 + 2(1\ldots. + 2(1+2))\ldots..)$$

Evaluate: The group with the least steps is technically the winner. All students will solve 2 problems as an exit slip for leaving the room. These problems are:

$3x - 4 = 20$ answer is 8

$2x + 24 = 6x - 8$ answer is 7

Vertical Strand: Algebra – Patterns, Relations, Functions and Their Properties & Attributes

7.5A use concrete and pictorial models to solve equations and use symbols to record the actions

A.7B investigate methods for methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, select a method, and solve the equations and inequalities **T**

A.7C interpret and determine the reasonableness of solutions to linear equations and inequalities $\mathbf{T}_{\mathbf{9,10,exit}}$

G.7C derive and use formulas involving length, slope, and midpoint

2A.2A use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations;

2A.3A analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems;

2A.3C interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts;

P.3D use properties of functions to analyze and solve problems and make predictions

Optimizing Equations

Mr. Ojeda

- Review of what I do

Task:

- What is the fastest way to get dressed in the morning?

Solving equations requires steps.

- What is the first step in this equation? $5x + 3 = 3x + 9$

$$5x - 3x + 3 = 3x - 3x + 9$$

$$2x + 3 = 9$$

$$2x + 3 - 3 = 9 - 3$$

$$2x = 6$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

Now let's solve using nsolve.

$5x + 3 = 3x + 9$

Now let's solve it using nsolve.

$$\frac{3x}{4} + 7 = 10$$

Now practice:

This is what I do: $\sum_{n=0}^{30} 2^n$

$= 2^0 + 2^1 + 2^2 + 2^3 + \ldots. + 2^{30}$

SLOW!!! because of the repeated multiplication

$= 1 + 2(1 + 2^1 + 2^2 + \ldots + 2^{29})$

$= 1 + 2(1 + 2(+ 1 + 2(1\ldots. + 2(1+2))\ldots..)$

Lesson 2

Reflections: This lesson was really engaging for students and very effective in transmitting the research concept to the students. In the first part the students get to think about real life activities that they normally do. This is smoothly connected to the mathematics that the students will be doing while solving equations, it should be clear to the students that taking steps in order is important in order to find their solution. Then they learn how to use the nsolve() function in the calculator. With the function they don't have to go through the steps of solving an equation which helps them get it done faster. After this they practice solving equations with the calculator which makes it very easy for longer, tedious problems. After they are finished with the worksheet, the power point gives an example that is considered to be an optimization that is not very complicated for them to understand. The example in the presentation goes all the way to 2^30 but we made it easier by explaining examples for 2^3 first.

This lesson is designed mostly for freshmen taking Algebra 1 which makes it hard when it comes to group activities. This lesson doesn't have a group activity contrary to other lessons but it seems to still be very engaging just because the students really understand what they have to do with the explanation given at the beginning.

One flaw that the lesson might have is that of students relying on technology too much when doing mathematics. This lesson is meant to teach them the concepts of solving equations as well as the advantage of technology and I believe they get to understand that without the analytical way of solving the equation there would never be way to do it faster.

Teacher Notes:

Student Worksheets / Handouts / Powerpoint	Charts to record the teams' steps, icons, powerpoint presentation to close the lesson, exit slip

KEY

Name:________________________________ Date:_________________________ Period:_____

Solve the Following Equations

Use the nsolve button to find the solutions.

1. $3(2x+5)=39$ $x=4$
2. $2(6k-1)=-38$ $x=-3$
3. $8(7-y)=-24$ $x=10$
4. $-4(8+5n)=8$ $x=-2$
5. $6(3x-5)-7x=25$ $x=5$
6. $-2(5+6m)+16=-90$ $x=8$
7. $15(t+2)+9t=6$ $x=-1$
8. $7w-3(4w+8)=11$ $x=-7$

Use the nsolve button to find the solutions, but then find the required additional answer.

9. $22-5(6v-1)=-63$ **Now find $2v+25$.** 31
10. $18x-(8x-7)=67$ **Now find $-3x-6$.** -24
11. $8(-2x-4)+12=-52$ **Now find $5x+2$.** 12
12. $2(9n-1)+7(n+6)=-60$ **Now find $6n-1$.** -25
13. $-3(3x+15)-(10+x)=35$ **Now find $-2x-1$.** 17
14. $11(4-6y)+5(13y+1)=9$ **Now find $5y+7$.** 207

Substitute in the number for the variable and then solve using the nsolve button. Many of your answers will be fractions!

15.	$3x - 5y = 20$,	given $(4, y)$	$\frac{8}{-5}$	-1.6
16.	$2x + 3y = 15$,	given $(x, 2)$	$\frac{9}{2}$	4.5
17.	$-2x - 5y = 11$	given $(1, y)$	$\frac{13}{-5}$	-2.6
18.	$x - y = -8$	given $(6, y)$	14	
19.	$9x + 6y = -50$	given $(x, 2)$	$\frac{-62}{9}$	-6.89
20.	$-3x + 5y = 25$	given $(x, -3)$	$\frac{40}{-3}$	$-13.33...$
21.	$y = 5x - 8$	given $(x, 4)$	$\frac{12}{5}$	2.4
22.	$y = -3x - 6$	given $(2, y)$	-12	
23.	$y = 10x + 7$	given $(7, y)$	77	
24.	$y = -6x + 9$	given $(x, 3)$	1	

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16. $2x + 3y = 15$, given $(x, 2)$

17. $-2x - 5y = 11$ given $(1, y)$

18. $x - y = -8$ given $(6, y)$

19. $9x + 6y = -50$ given $(x, 2)$

20. $-3x + 5y = 25$ given $(x, -3)$

21. $y = 5x - 8$ given $(x, 4)$

22. $y = -3x - 6$ given $(2, y)$

23. $y = 10x + 7$ given $(7, y)$

24. $y = -6x + 9$ given $(x, 3)$