NSF GK-12 MAVS Project Lesson 6

GK-12 MAVS Fellow: Larrissa Owens

GK-12 MAVS Mentor Teacher: Daree Yancey

Title of Lesson: Exponential What?

Class: 7th Grade Math Pre-Algebra

Topic: Exponential Growth and Decay

Objectives: Through the use of a penny activity to model a half-life trend, students will be able to better understand scenarios that exhibit exponential growth or decay. Students will extend what they know about linear functions to this new type of non-linear function. In a follow up activity graphing calculators will be used as a tool as students discover how the base of an exponential function effects the graph, they will further be able to comment on y-intercepts, domain, range, and be able to form a story to accurately describe what is occurring during exponential decay.

Standards TEKS: A.11C Analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods, (A2. 11F) determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods; analyze a situation modeled by an exponential function, PC 3.B Use functions such as exponential to model real life data.
Standards NCTM - 2. Algebra, 6. Problem Solving, 8. Communication, 9. Connections, 10. Representation.

Key vocabulary: Exponential growth and decay, apoptosis, y-intercept, half-life

Materials and Resources: Graphing Calculators, 50 Pennies (per group), PowerPoint, Penny Experiment worksheets, Page 2 Penny Experiment Worksheet, Communicator sleeves, white board markers, communicator worksheet, worksheet lesson 6b

Research Setting/Connection/Motivation: Cell apoptosis (death) and cell proliferation (cell splitting) both play important roles in modeling that we do of cell population involved in immune response. Specifically the final graph that the students will plot and investigate is an isolation of the cell death of macrophages from our actual model. Other than the slight change to the starting value, the graph is accurate to what we work with as we investigate the effects of immune response. The exponential decay has been observed in biology labs and is an agreed upon method for mathematically modeling population growth (not only limited to cell populations).

Prior knowledge: Linear equations, y-intercept, domain, range, exponents

Vertical Strands: TEKS-A.11C Analyze data and represent situations involving exponential growth and decay using concrete models, tables, graphs, or algebraic methods, (A2. 11F) determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods; analyze a situation modeled by an exponential function, PC 3.B Use functions such as exponential to model real life data. College Algebra: exponential functions and affects of base change of exponential function graphs. Statistics: knowledge of scatterplot and exponential regression functions.

Math Journal

Wednesday: Explain whether the following is a function or is not a function:

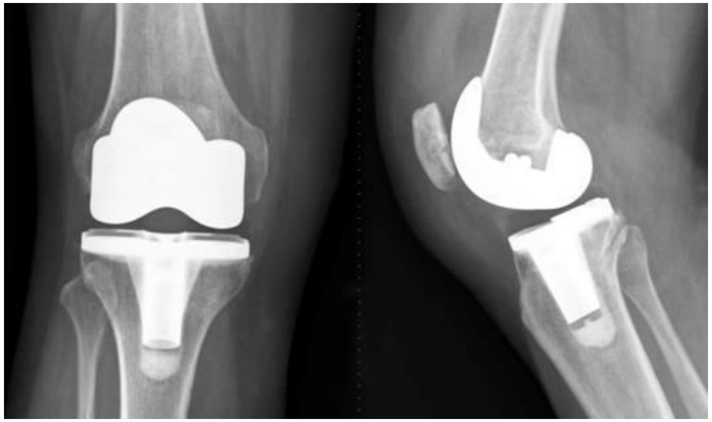
Number of hours studied to the score on an exam

Miles over the speed limit and amount of the ticket.

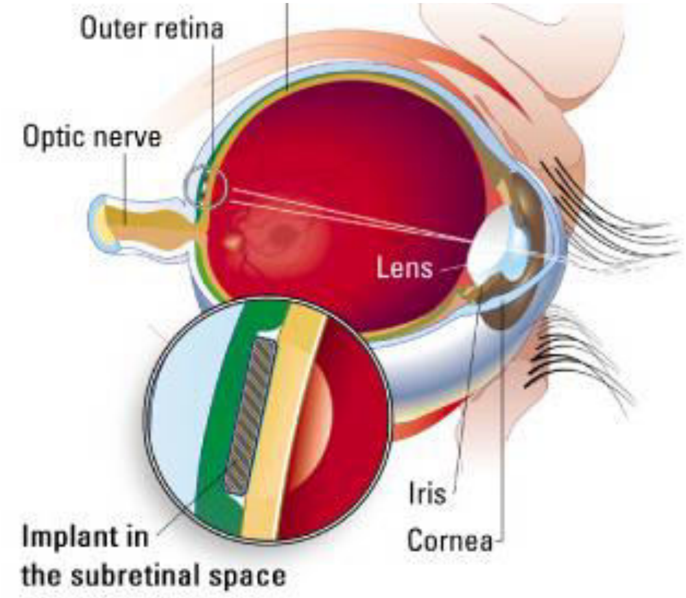
Larrissa Owens

Graduate Students, GK12 Fellow

Department of Mathematics, UTA

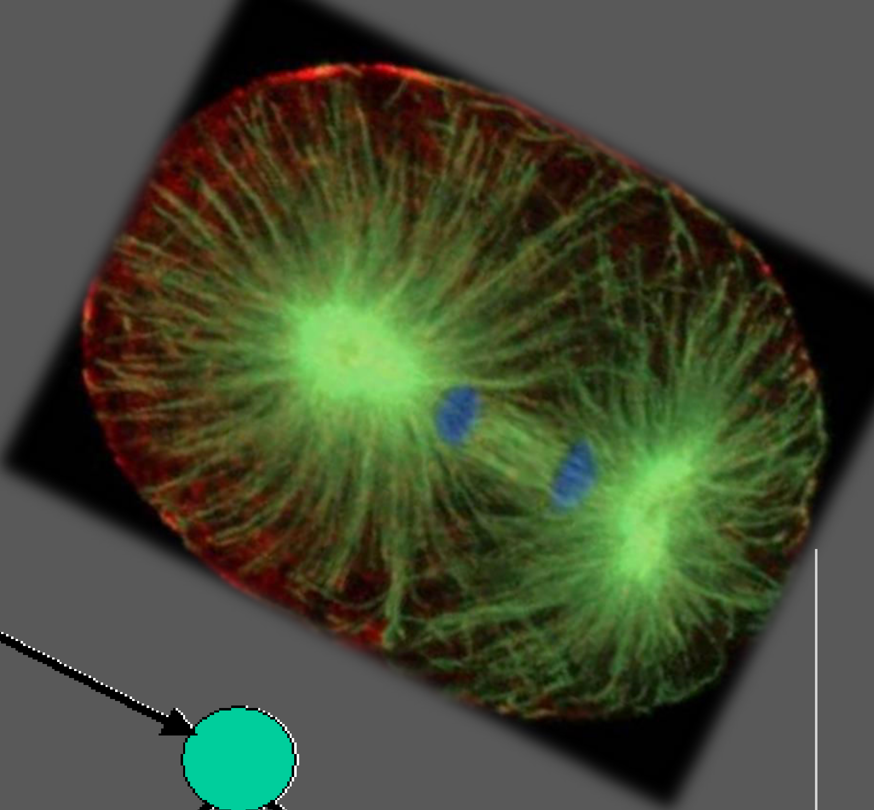


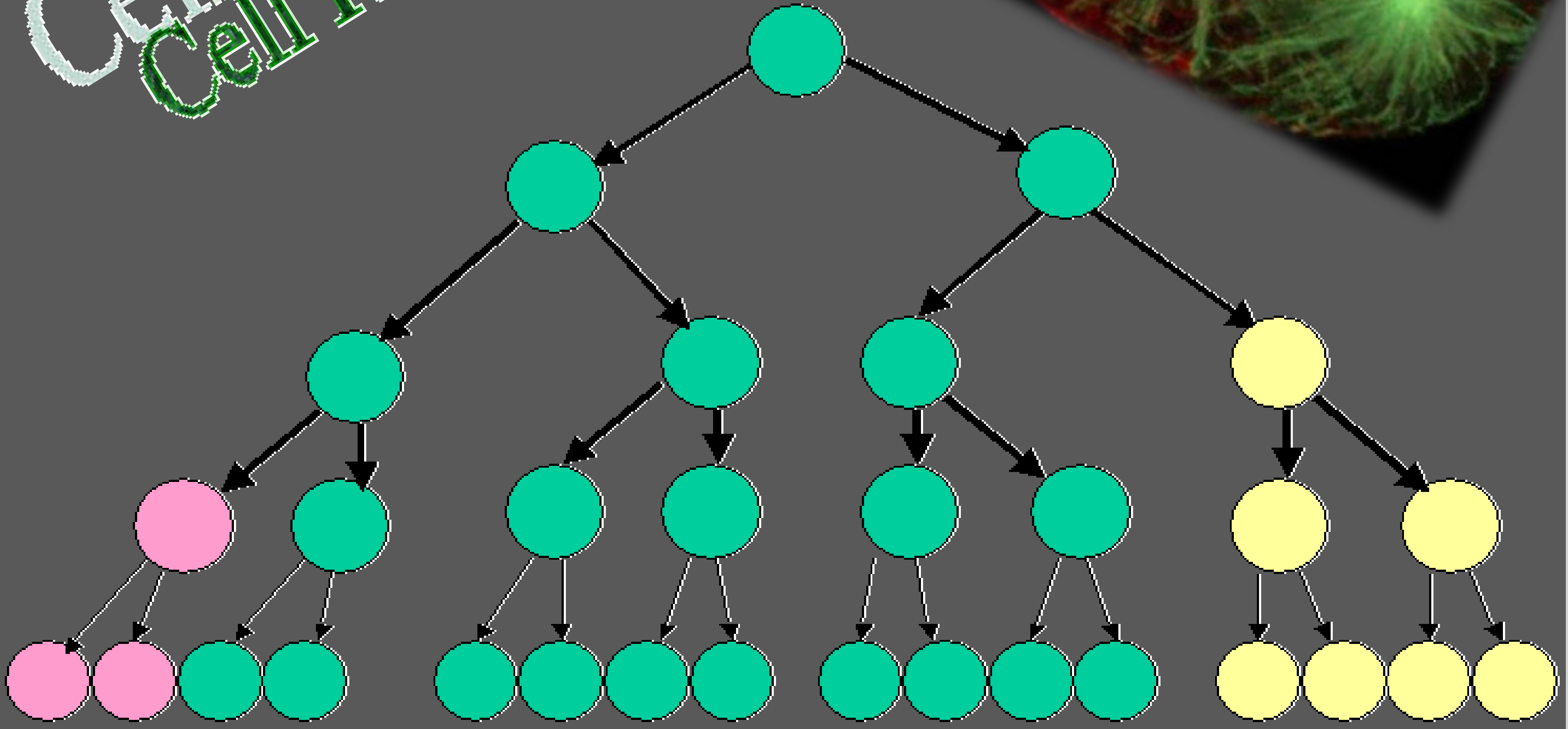
SIGHTS SET ON ADVANCING TREATMENT FOR RETINAL DISEASE



Cell Replication

Cell Replication





If you were to graph the Cell Replication what do you think it would look like?

If it replicates once each hour then you have:

Hour (t)	# of cells (y)
0	1
1	2
2	4
3	8
4	16

What is the y-intercept?

$$y = (?)t+1$$

Is there a constant value that you are multiplying the hour (independent variable) by where adding one to the result would give you the corresponding number of cells (the dependent variable)?

If you were to graph the Cell Replication what do you think it would look like?

If it replicates once each hour then you have:

Hour (t)	# of cells (y)
0	1
1	2
2	4
3	8
4	16

On your communicator, give a quick sketch of what you think the graph of the # of cells will look like over time.

If you were to graph the Cell Replication what do you think it would look like?

If it replicates once each hour then you have:

Hour (t)	# of cells (y)
0	1
1	2
2	4
3	8
4	16

Is there a constant value that you are multiplying the hour (independent variable) by where adding one to the result would give you the corresponding number of cells (the dependent variable)?

Linear Equation: $y = (?)t+1$

What do you think the equation of this graph might involve? Do you see any pattern in the column of cell amounts?

Hour (t)	# of cells (y)
0	1
1	2
2	4
3	8
4	16

What do you think the equation of this graph might involve? Do you see any pattern in the column of cell amounts?

Hour (t)	# of cells (y)
0	$1 = 2^0$
1	$2 = 2^1$
2	$4 = 2^2$
3	$8 = 2^3$
4	$16 = 2^4$

$$y = a\,b^t$$

What is the value of “a” for our model of cell replication ?

What is the value of “b” for our model of cell replication?

What do you think the equation of this graph might involve? Do you see any pattern in the column of cell amounts?

Hour (t)	# of cells (y)
0	$1 = 2^0$
1	$2 = 2^1$
2	$4 = 2^2$
3	$8 = 2^3$
4	$16 = 2^4$

$$y = 1 \text{ x } 2^{t}$$

What is the value of “a” for our model of cell replication ?

What is the value of “b” for our model of cell replication?

$$y = a\,b^t$$

What is the “t” referred to as in this equation?

$$y = a b^t$$

What is the “t” referred to as in this equation?

exponent

$$y = a b^t$$

What is the “t” referred to as in this equation? → What do you think they named equations of this form as a result?

exponent →

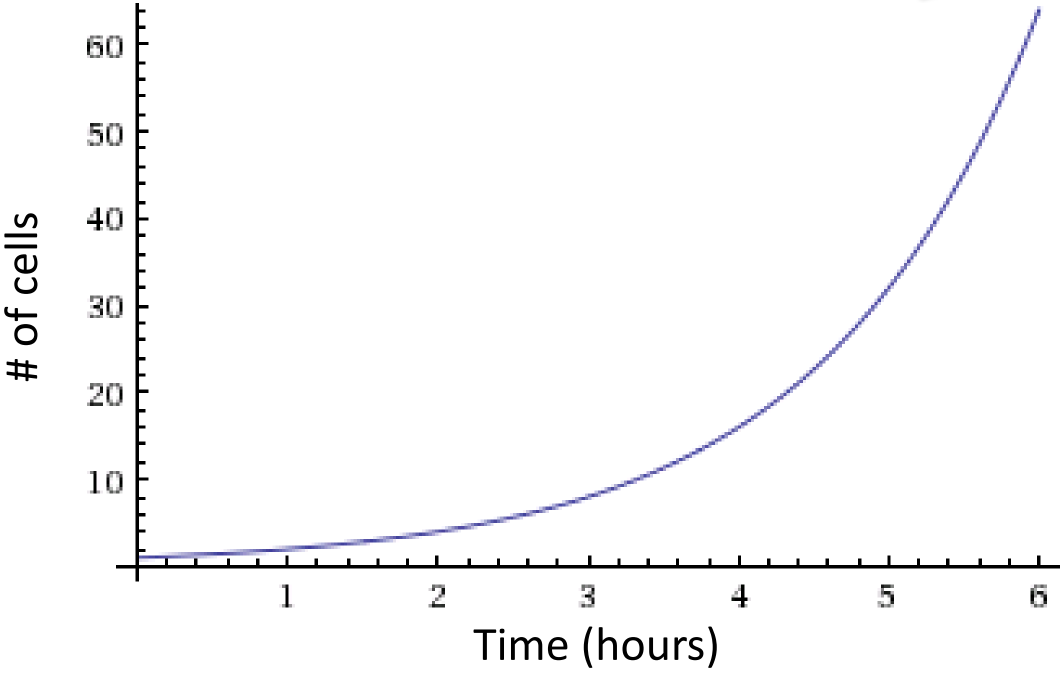
$$y = a b^t$$

What is the “t” referred to as in this equation? → What do you think they named equations of this form as a result?

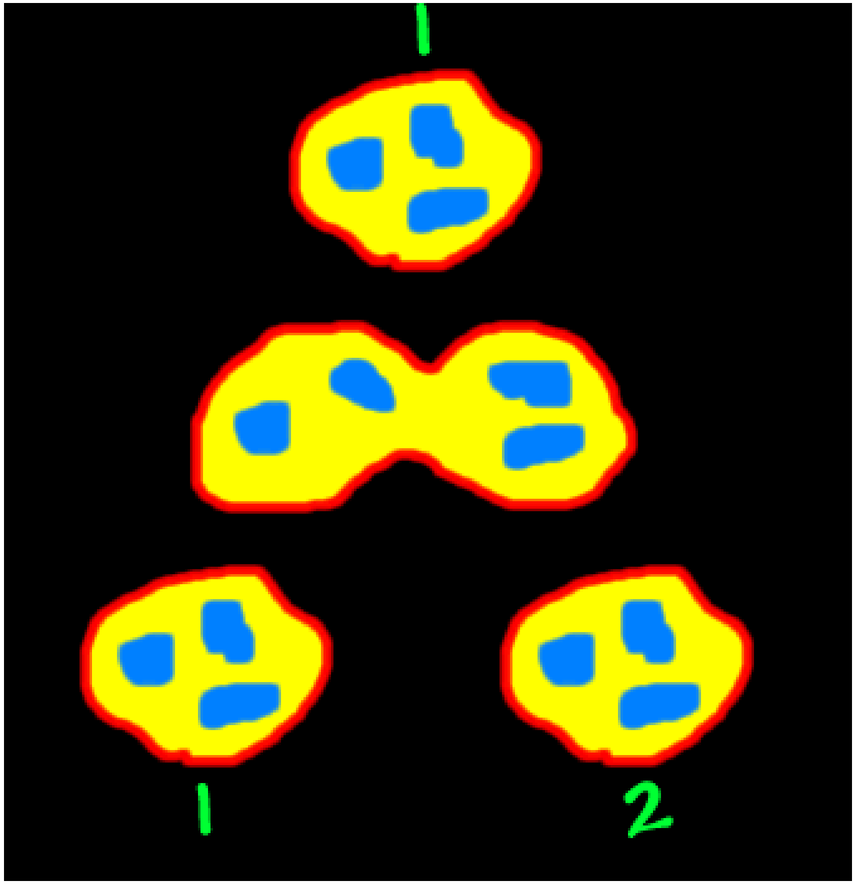
exponent → **exponential**

Exponential Growth

$y = 2^t$



Pop Quiz



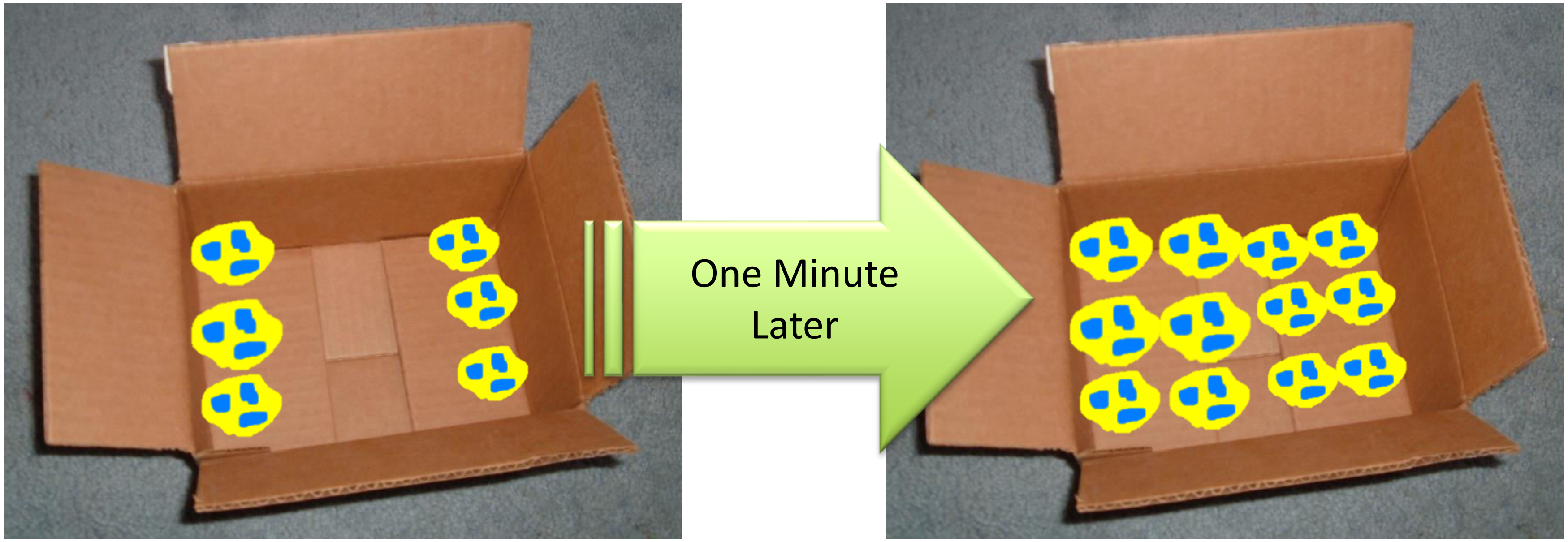
Assume: every minute a cells splits to form 2 new cells.

At 5pm you put one cell into a box.

At 6pm the box first reaches full capacity.

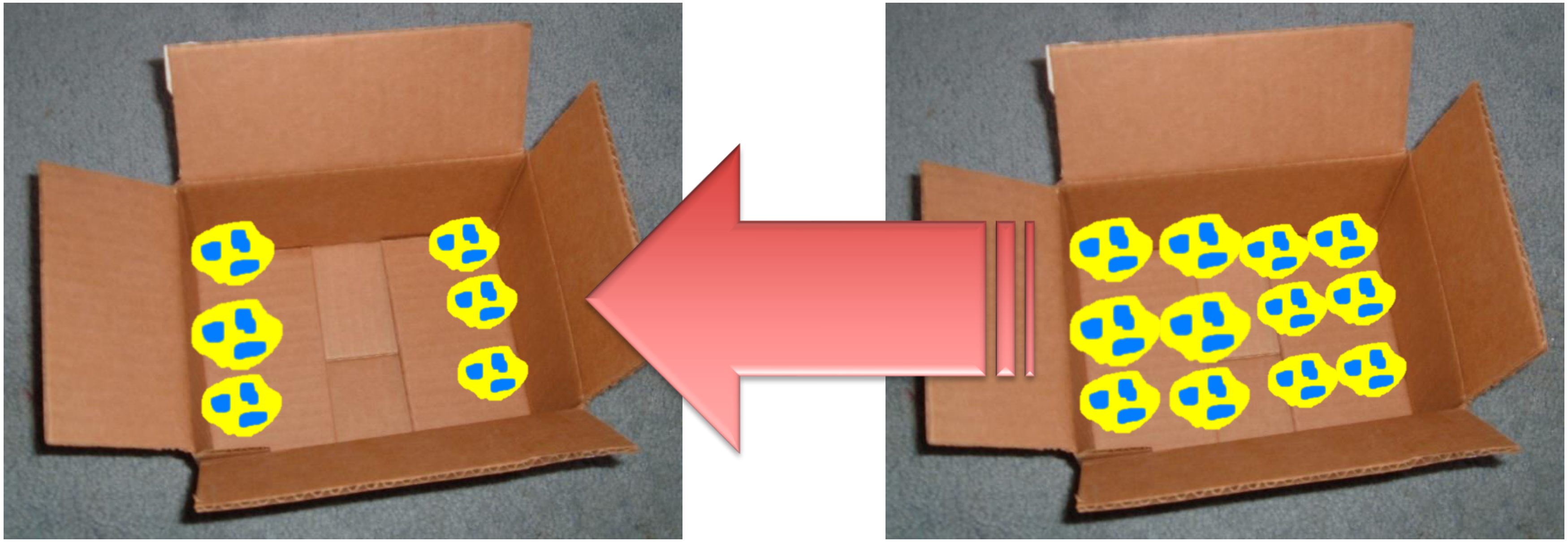
When was the box half full?

5:59



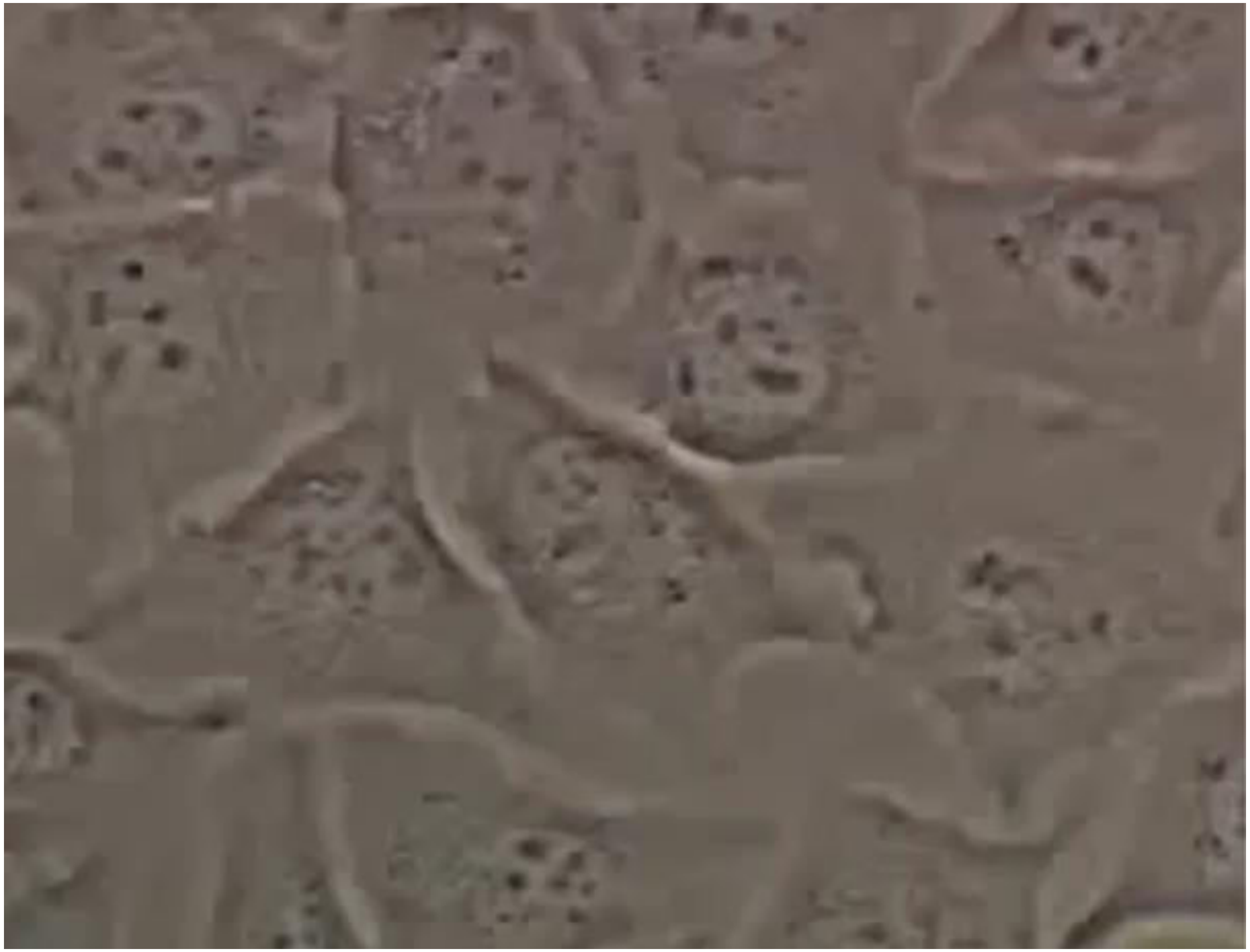
One Minute Later

Exponential Decay



Recall

Apoptosis:
Programed cell death



Half Life: Penny activity

What do you think the equation of this graph might involve? Do you see any pattern in the column of cell amounts?

Hour (t)	# of cells (y)
0	16
1	8
2	4
3	2
4	1

$$y = a\,b^{t}$$

What is the value of "a" for our model of cell replication ?

What is the value of "b" for our model of cell replication?

What do you think the equation of this graph might involve? Do you see any pattern in the column of cell amounts?

Hour (t)	# of cells (y)	
0	16 = 16	$=16 \times .5^0$
1	8 = 16 x .5	$=16 \times .5^1$
2	4 = 16 x .5 x .5	$=16 \times .5^2$
3	2 = 16 x .5 x .5 x .5	$=16 \times .5^3$
4	1 = 16 x .5 x .5 x .5 x .5	$=16 \times .5^4$

$$y = 16\,(0.5)^t$$

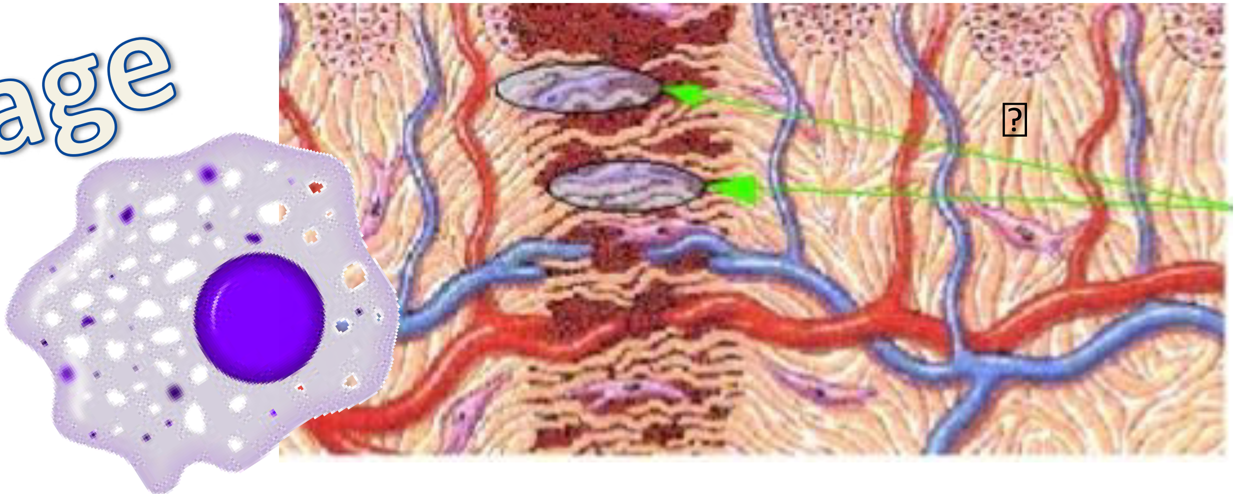
We start with 16

We divide our populationin half after each hour?

Clearing out your calculator:

- Press **Y=** and hit **CLEAR**
- Press **2nd** and **+** then press **4** and **enter** . The screen Should say “Clr All Lists” then **Enter** one more time

Macrophage



Isolating just the decay term of the macrophages:

$$\frac{dD}{dt} = -f_0\lambda_1 MD + f_0\lambda_3 M$$

$$\frac{dC}{dt} = f_1 D + f_2\lambda_3 M - f_3\lambda_2 MC - f_4 C$$

$$\frac{dF}{dt} = a_{12} FC + a_2 F\left(1 - \frac{F}{F_0}\right) - a_3 F$$

$$\boxed{\frac{dM}{dt} = a_{11} MCH(M_0 - M) - a_0 M}$$

$$\frac{dE}{dt} = a_{16} F\left(1 - \frac{E}{E_0}\right)$$

$$\frac{dM}{dt} = a_{11} MCH(M_0 - M)\boxed{- a_0 M}$$

$$\boxed{-a_0 M}$$

Isolating just the decay term of the macrophages, we get the equation

$$M=(\text{initial amount}) \times e^{-0.045t}$$

e =2.718281828459045235360287471352...

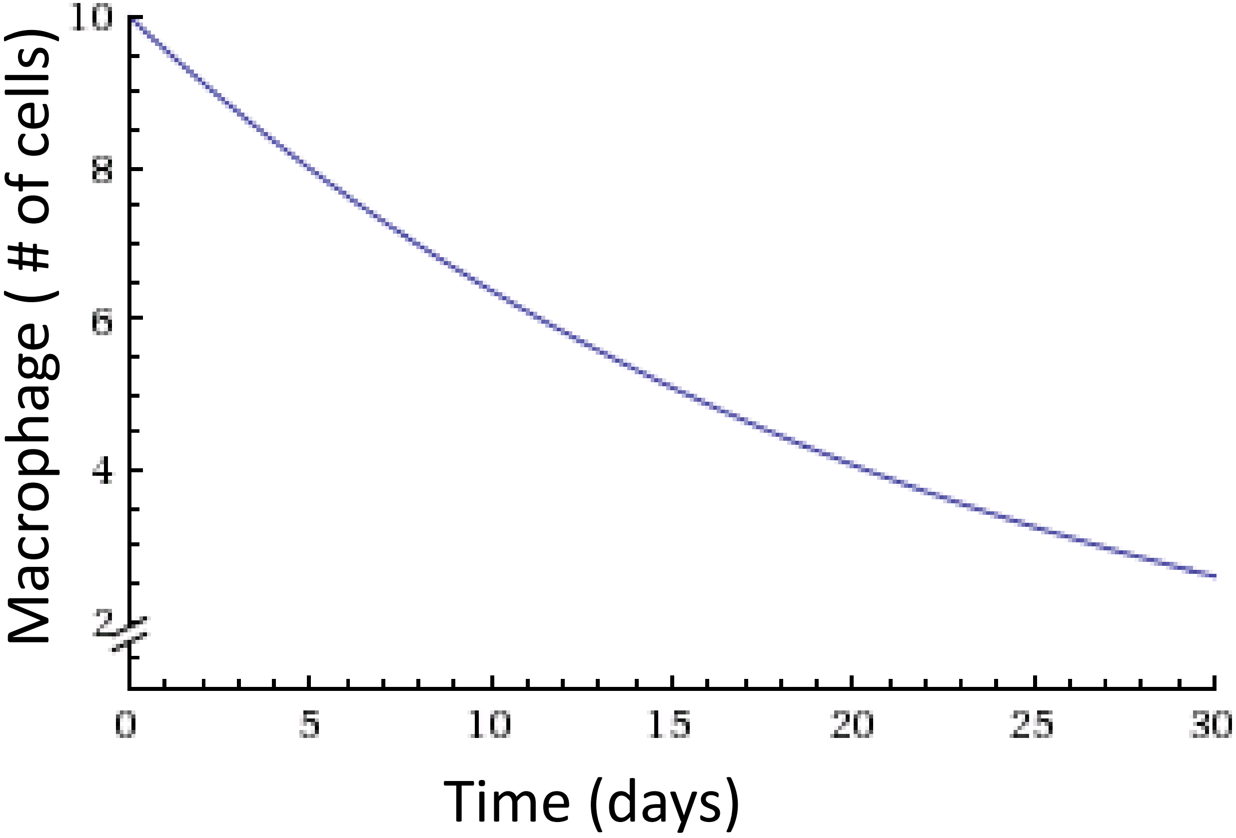
Quick Review: What type of number is e, rational or irrational?

If we start with 10 macrophages then our equation for modeling the growth is:

$$M=10 \times (2.718)^{-0.045t}$$

If we start with 10 macrophages then our equation for modeling the growth is:

$$M=10 \times (2.718)^{-0.045t}$$



Lesson Presentation:

1. **Engage:** Warm up reviewing the previous day's topic of functions, students then plot points on their communicator sleeves and struggle to fit the data into their knowledge of equations (limited to linear equations)
2. **Explore:** Students are introduced to exponential functions and work through a penny activity to better grasp Half –Life trends
3. **Explain:** Students will describe how the base of the exponential affects the graph.
4. **Extend:** Students will see how exponential growth and decay graphs again the following week and will investigate multiple exponential graphs and be able to describe how the base of the exponential affects the graph.
5. **Evaluate:** Students will be presented with the "pop quiz" question during the PowerPoint to determine if the class as a whole is beginning to understand the difference between linear and non-linear functions. Furthermore students will be asked to figure out the exponential equation for half-life during the penny activity.

Teacher Notes:

Prior to this lesson students had been working regularly with linear functions but had never seen exponential functions. The PowerPoint aims to give the students a non-linear set of data and asks them to figure out a linear equation that fits perfectly with the data (not a possible task, but still constructive). This allows students to realize that linear slope-intercept equations won't work for every table of data, it also provides a good launching point as students recognize new patterns and ways to express those in equations that are different from the forms they have previously seen. The communicators help the students to have a way to sketch out this process as they begin to realize that cell growth is not a linear trend and must instead be modeled using an equation in which the independent variable resides in the exponent (the PowerPoint indicated at which point the communicators should be used). Students are presented with the "pop quiz" question during the PowerPoint to help further emphasize the concept of exponential growth and just how counter intuitive some of it's properties are since they are so used to thinking linearly. Most students think the box should be half full at 5:30, a correct assumption if the growth was linear (which is still an answer with applauding), it took most classes several guesses (and took me explaining that if I have 1 full box at 6:00 then I'd have 2 full boxes at 6:01) before they finally started to really grasp the exponential trend and reached the correct solution.

The students really benefited from verbal instructions leading them through the penny experiment. When they were led through the calculator steps then the lesson could be finished in 45 min, otherwise a longer time was needed to help students troubleshoot when they didn't follow a written instruction correctly. Students really enjoyed this activity and really started to understand what half-life looks like. The power point has a slide which instructs the students to clear out their calculators, this is key so that data doesn't interfere with the next class period. During the final class of the day you may also want them to turn the Stat Plot back off so that the calculators are ready for the extension activity where they will only be graphing from the Y= window.

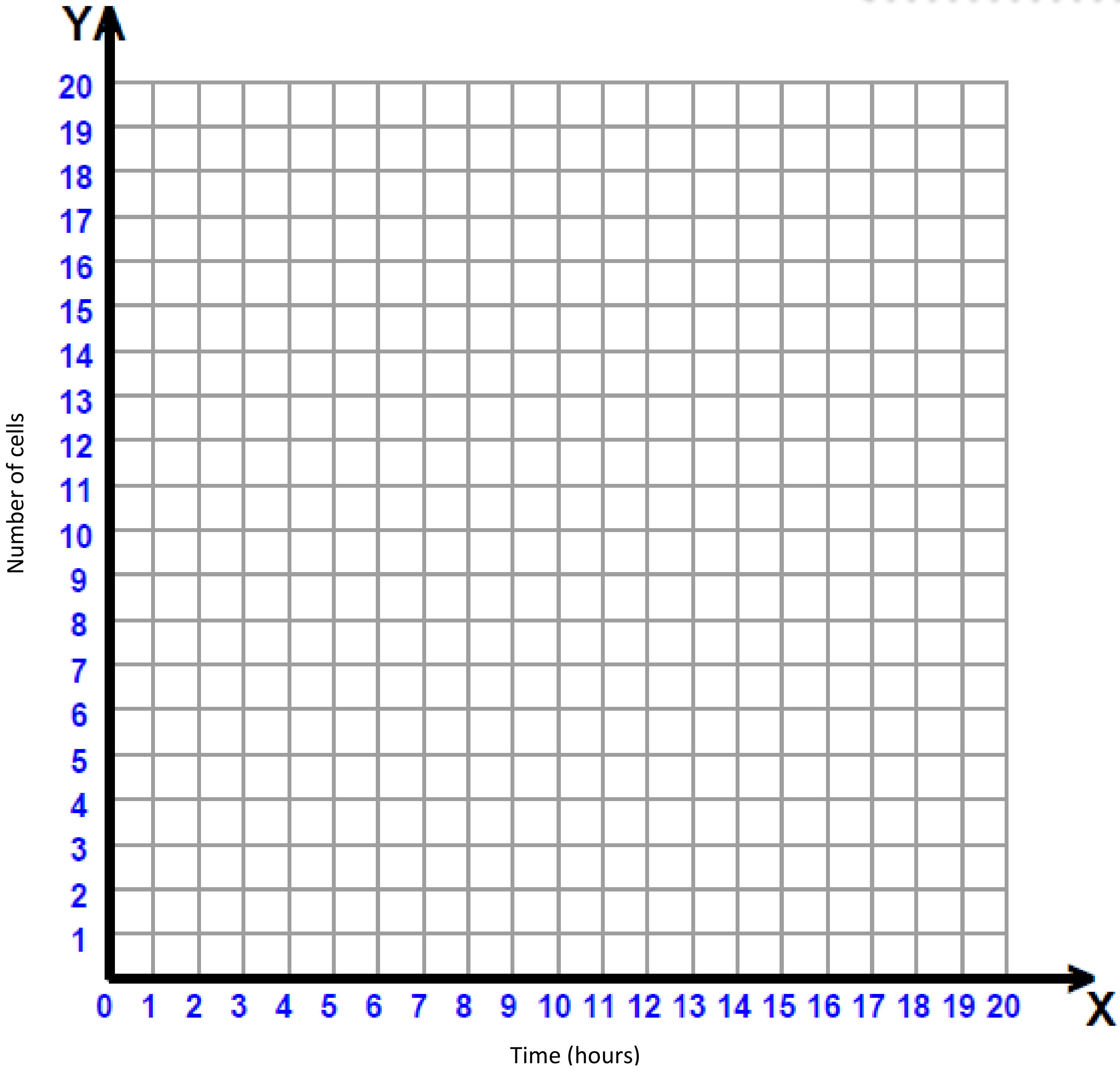
The extension activity was a great opportunity for students to better connect the features of exponential equations with their corresponding behaviors. During the calculator assisted exploration the students were able to see the shift from exponential growth to decay when the base term went from being greater than one to being less than one. Some made the false generation that increasing functions must have a whole number base, but after providing them with the example y=(5/3)^x to try they soon understood that it is the quantity being greater than one not the fact that it is a whole number that produced the increasing quality in the graph.

Graphing Cell Replication

If a cell replicates once each hour then you have:

Hour (t)	# of cells (y)
0	1
1	2
2	4
3	8
4	16

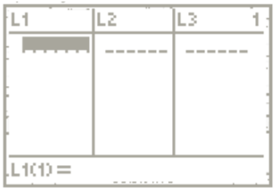
What is the y-intercept?



Name_____________KEY___________________

Entering Penny Data in TI-83 Graphing Calculator

Step 6: Press STAT ; Press 1 (Edit) and press enter. Begin entering your data from your experiment in L_1 (toss#) and in L_2 (pennies remaining).*Any entry where no pennies remained should be left out.



```
L1      L2      L3     1
------  ------  ------

L1(1) =
```

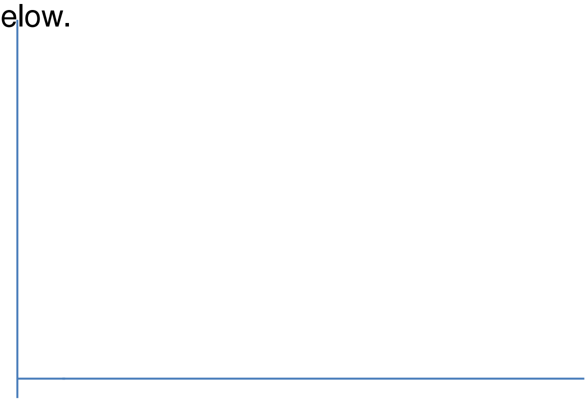
then

Step 7: Press 2nd STAT PLOT ; Press 1

(Plot 1); Put cursor on ON and press enter.

```
Plot1  Plot2  Plot3
On  Off
Type:
Xlist:L1
Ylist:L2
Mark: ▫  +  ·
```

Step 8: Press ZOOM ; Press 9 (ZoomStat). Draw the graph below.



Step 9: To find the best fit graph: Press STAT ; Press CALC ; Press 0 (ExpReg); The word "ExpReg" will appear on your screen. Press ENTER . You will see the following screen:

```
ExpReg
 y=a*b^x
 a=
 b=
```

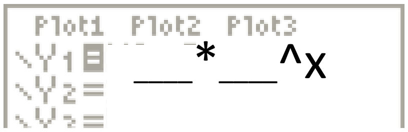
- This equation is the exponential function that best fits your data. Round a and b to two decimal places and record the equation here:

$y = \_\_\_\_\_\_\_\_(\_\_\_\_\_\_\_)^x$

- What mathematical equation is more commonly used to model the half-life of 50 cells?

$y = \_50\_(\_.5\_)^x$

Step 10: Press Y=; type this equation in and press GRAPH to see how close your data points are to this equation. Add the exponential decay line to your graph above in Step 8.



```
Plot1  Plot2  Plot3
\Y1 = ___*___^x
\Y2 =
\Y3 =
```

How closely did your experiment fit with the commonly used decay equation?

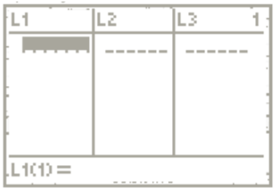
____

Images and activity inspired by: http://www.nsa.gov/academia/_files/collected_learning/high_school/algebra/exploring_exponential_growth.pdf

Name________________________________

Entering Penny Data in TI-83 Graphing Calculator

Step 6: Press STAT ; Press 1 (Edit) and press enter. Begin entering your data from your experiment in L_1 (toss#) and then in L_2 (pennies remaining).*Any entry where no pennies remained should be left out.


L1 L2 L3 1

L1(1) =

Step 7: Press 2nd STAT PLOT ; Press 1 (Plot 1); Put cursor on ON and press enter.


Plot1 Plot2 Plot3

On Off

Type:

Xlist:L1

Ylist:L2

Mark: ▫ + ·

Step 8: Press ZOOM ; Press 9 (ZoomStat). Draw the graph below.

Step 9: To find the best fit graph: Press STAT ; Press CALC ; Press 0 (ExpReg); The word "ExpReg" will appear on your screen. Press ENTER . You will see the following screen:


ExpReg

y=a*b^x

a=

b=

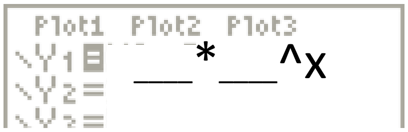
- This equation is the exponential function that best fits your data. Round a and b to two decimal places and record the equation here:

y =________(______)^x

- What mathematical equation is more commonly used to model the half-life of 50 cells?

y =_____(_____)^x

Step 10: Press Y=; type this equation in and press GRAPH to see how close your data points are to this equation. Add the exponential decay line to your graph above in Step 8.


Plot1 Plot2 Plot3

\Y1= ___*___^x

\Y2=

\Y3=

How closely did your experiment fit with the commonly used decay equation? ____

Images and activity inspired by: http://www.nsa.gov/academia/_files/collected_learning/high_school/algebra/exploring_exponential_growth.pdf

Name____

Penny Experiment: Exponential Decay

Step 1: In your group place all 50 pennies in your cup.

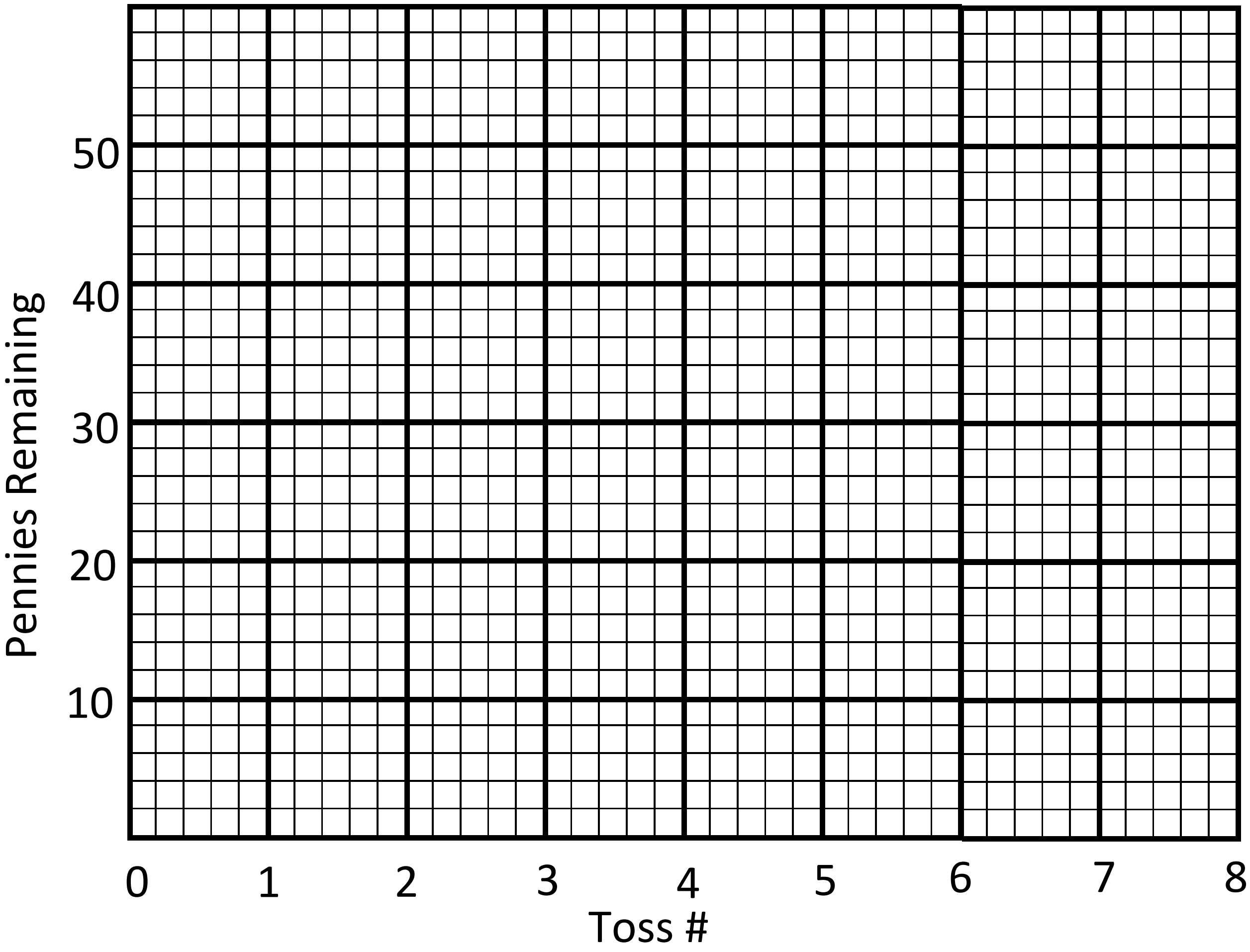
Step 2: Shake them around really well and pour them out onto the table.

Step 3: Remove all pennies that were tails and set them aside, count and record the number of remaining pennies in the table below and place them back in your cup.

Step 4: Repeat steps 2 and 3 until no pennies remain.

Toss #	Pennies Remaining
0	50
1	
2	
3	
4	
5	
6	
7	
8	

Step 5: Plot your experimental results below.



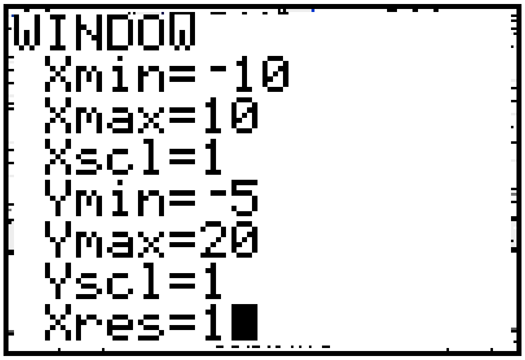
Name ________________________________ Period_______

(Pages 1-3 published by Sharon Nussbaum, Jessica Redlin, and Denise Stewart at http://www.nsa.gov/academia/_files/collected_learning/high_school/algebra/exploring_exponential_growth.pdf, page 4 is unique to this lesson)

Activity 1 $y = ab^x$

Use your TI-83 graphing calculator to graph the following functions and answer the questions.

Suggested Window:



WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-5
Ymax=20
Yscl=1
Xres=1

A. Graph the following exponential equation

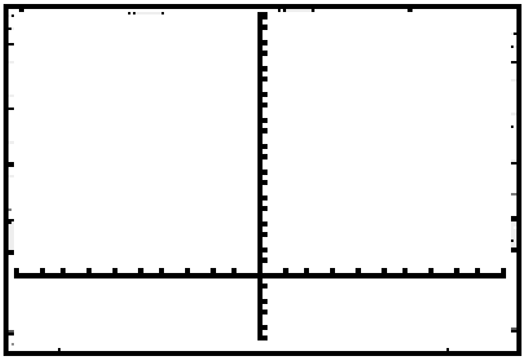
1. $y = 2^x$

a. y-intercept _________

b. increasing, decreasing or neither (circle one)

c. a = _________

d. b = _________



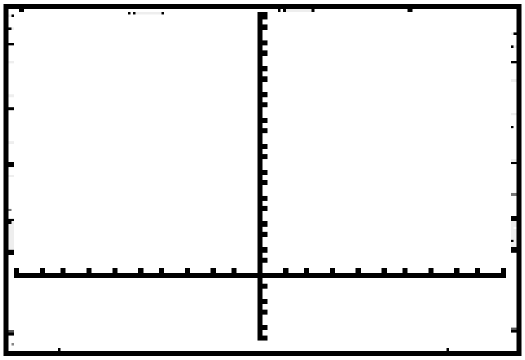
2. $y = 3^x$

a. y-intercept _________

b. increasing, decreasing, or neither (circle one)

c. a = _________

d. b = _________



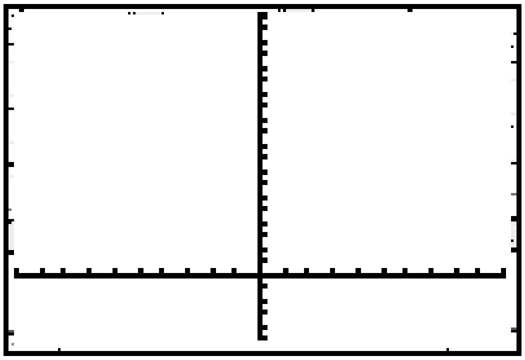
3. $y = 10^X$

a. y-intercept ________

b. increasing, decreasing, or neither (circle one)

c. a = ________

d. b = ________



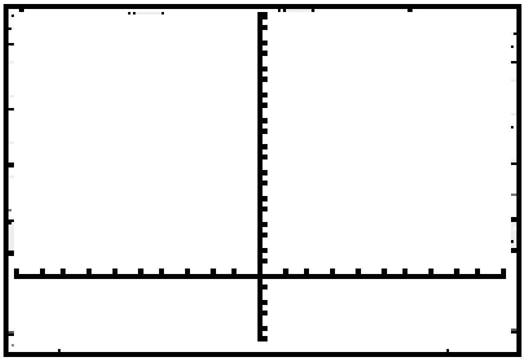
4. $y =1^X$

a. y-intercept ________

b. increasing, decreasing, or neither (circle one)

c. a = ________

d. b = ________



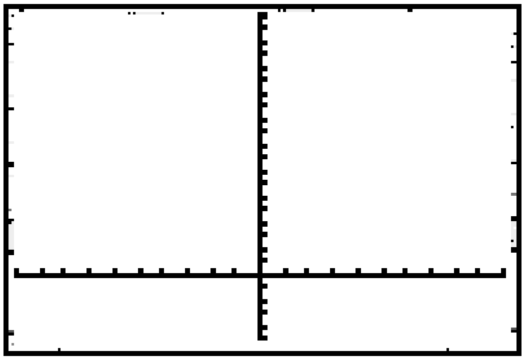
5. $y = (1/2)^X$

a. y-intercept ________

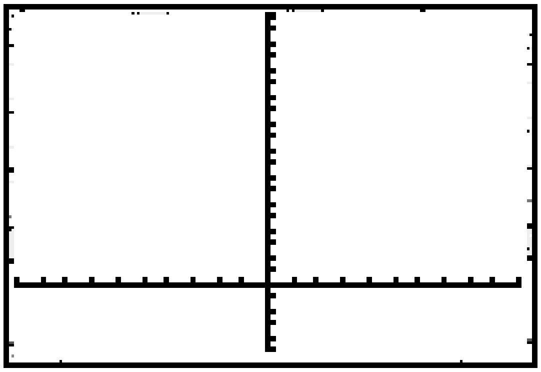
b. increasing, decreasing, or neither (circle one)

c. a = ________

d. b = ________



6. $y = (1/3)^x$



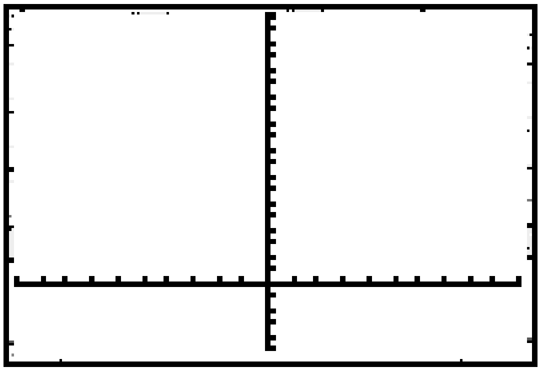
a. y-intercept ________

b. increasing, decreasing, or neither
(circle one)

c. a = ________

d. b = ________

7. $y = (1/10)^x$



a. y-intercept ________

b. increasing, decreasing, or neither
(circle one)

c. a = ________

d. b = ________

8. What point do they all have in common?__________

9. List the equations that are increasing.__________________________

____

10. List the equations that are decreasing.__________________________

____

11. When an exponential graph is increasing , it shows exponential growth . What are the similarities of all the equations that produce such graphs?
____
____

Name ________________________________ Period_______

Use your TI-83 graphing calculator to graph the following functions and answer the questions.

Suggested Window:

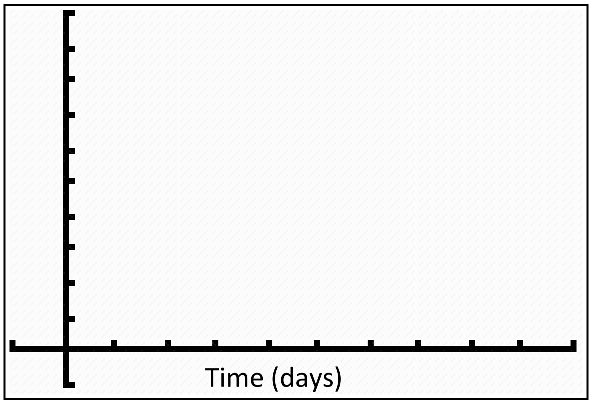
Xmin = 0
Xmax= 60
Xscl= 5
Ymin=0
Ymax=10
Yscl = 1
Yres= 1

12. Graph $y = 10(2.718)^{-0.045t}$

a. y-intercept ________________

b. (circle one) increasing, decreasing or neither

c. Complete the table: Round to the 3rd decimal place



x	y
0	
10	
20	
30	
40	
50	
60	

d. What does the y-intercept represent biologically?

____

e. The equation represents the decay rate of macrophage cells, how would you describe the behavior of the graph (i.e. is it decaying at a constant rate, is it decaying exponentially)?

____

f. Knowing that the macrophage population should return to near zero in order to imply that a person has healed, what would you conclude about this patient?

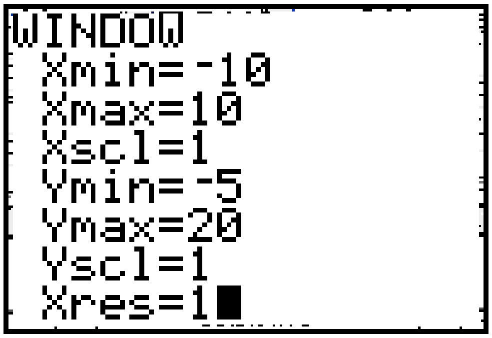
____

Name ___________KEY____________________ Period_______

(Pages 1-3 published by Sharon Nussbaum, Jessica Redlin, and Denise Stewart at http://www.nsa.gov/academia/_files/collected_learning/high_school/algebra/exploring_exponential_growth.pdf, page 4 is unique to this lesson)

Activity 1 $y = ab^x$ **KEY**

Use your TI-83 graphing calculator to graph the following functions and answer the questions.



```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-5
 Ymax=20
 Yscl=1
 Xres=1█
```

A. Graph the following exponential equation

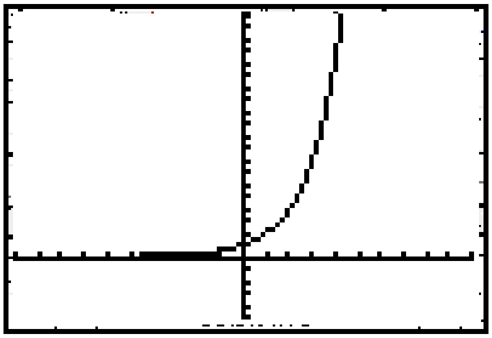
1. $y = 2^x$

a. y-intercept __(0, 1)__

b. increasing , decreasing, or neither (circle one)

c. a = ___1___

d. b = ___2___



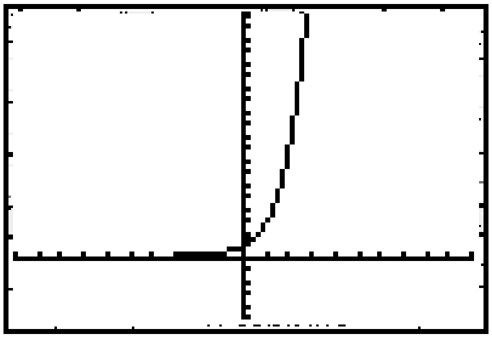
2. $y = 3^x$

a. y-intercept __(0, 1)__

b. increasing, decreasing or neither (circle one)

c. a = ___1___

d. b = ___3___



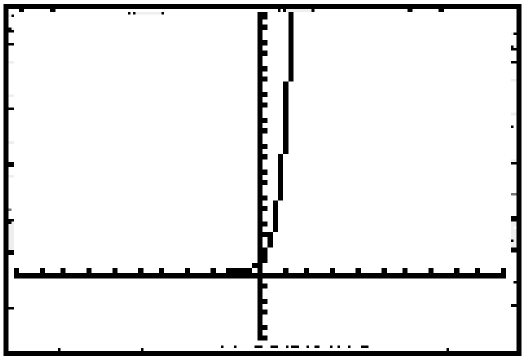
3. $y = 10^X$

a. y-intercept __(0, 1)__

b. increasing , decreasing, or neither (circle one)

c. a = ___1___

d. b = ___10___



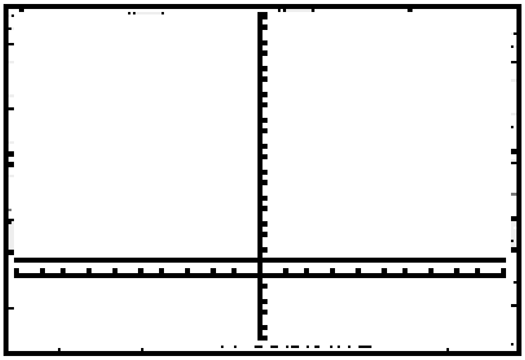
4. $y =1^X$

a. y-intercept __(0, 1)___

b. increasing, decreasing, or neither (circle one)

c. a = ___1___

d. b = ___1___



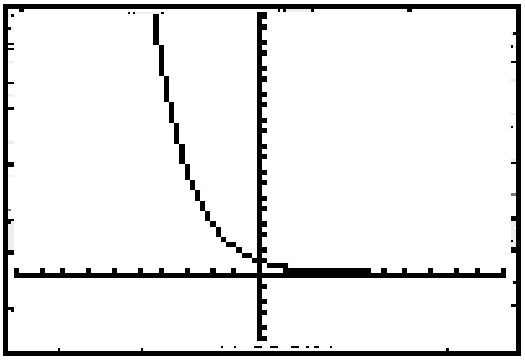
5. $y = (1/2)^X$

a. y-intercept __(0, 1)__

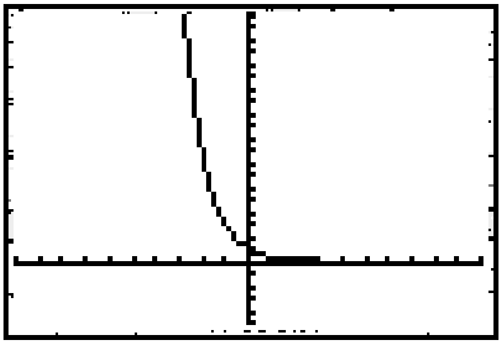
b. increasing, decreasing , or neither (circle one)

c. a = ___1___

d. b = ___1/2___



6. $y = (1/3)^x$



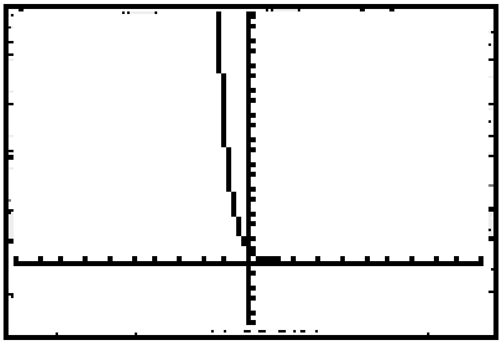
a. y-intercept __(0, 1)__

b. increasing, decreasing , or neither (circle one)

c. a = ___1___

d. b = ___1/3___

7. $y = (1/10)^x$



a. y-intercept __(0, 1)__

b. increasing, decreasing , or neither (circle one)

c. a = ___1___

d. b = ___1/10___

8. What point do they all have in common?__y-intercept _(0,1) __

9. List the equations that are increasing.___$y=2^x$, $y=3^x$, $y=10^x$______

____

10. List the equations that are decreasing._$y=(1/2)^x$, $y=(1/3)^x$, $y=(1/10)^x$

11. When an exponential graph is increasing , it shows exponential growth . What are the similarities of all the equations that produce such graphs?
__b>1___the base is greater than 1 ____________________________
____

Name _________________________________ Period_______

Use your TI-83 graphing calculator to graph the following functions and answer the questions.

Suggested Window:

Xmin = 0

Xmax= 60

Xscl= 5

Ymin=0

Ymax=10

Yscl = 1

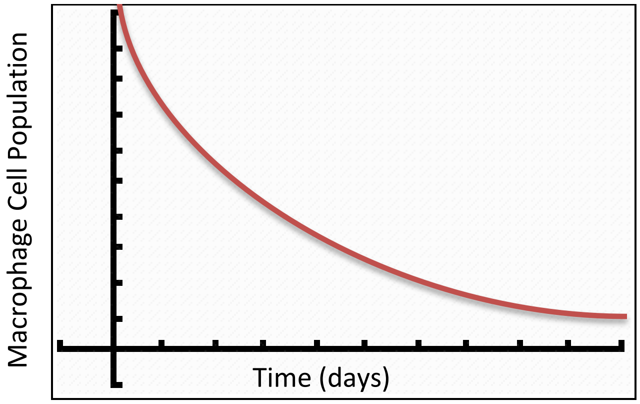
Yres= 1

12. Graph $y = 10(2.718)^{-0.045t}$

a. y-intercept _____(0,1)_______

b. (circle one) increasing, [✓ decreasing] or neither

c. Complete the table: Round to the 3rd decimal place



x	y
0	
10	
20	
30	
40	
50	
60	

d. What does the y-intercept represent biologically?

____How many macrophages we start with ______

e. The equation represents the decay rate of macrophage cells, how would you describe the behavior of the graph (i.e. is it decaying at a constant rate, is it decaying exponentially)? ___________

________decaying exponentially ______________

f. Knowing that the macrophage population should return to near zero in order to imply that a person has healed, what would you conclude about this patient? _________________

____________They are pretty close to healed after 60 days________________

Math Journal

Wednesday: Explain whether the following is a function or is not a function:

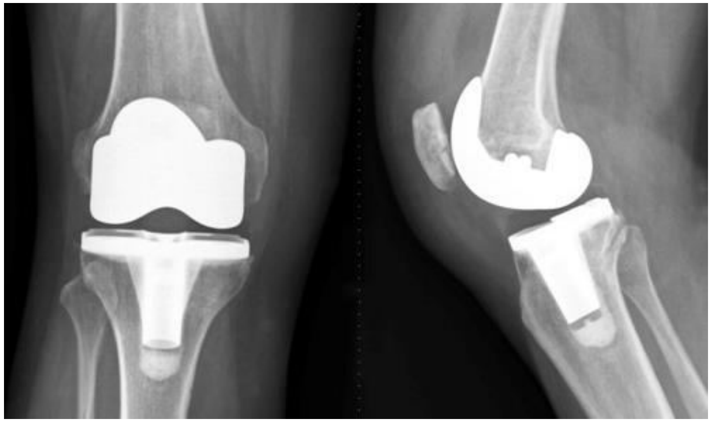
Number of hours studied to the score on an exam

Miles over the speed limit and amount of the ticket.

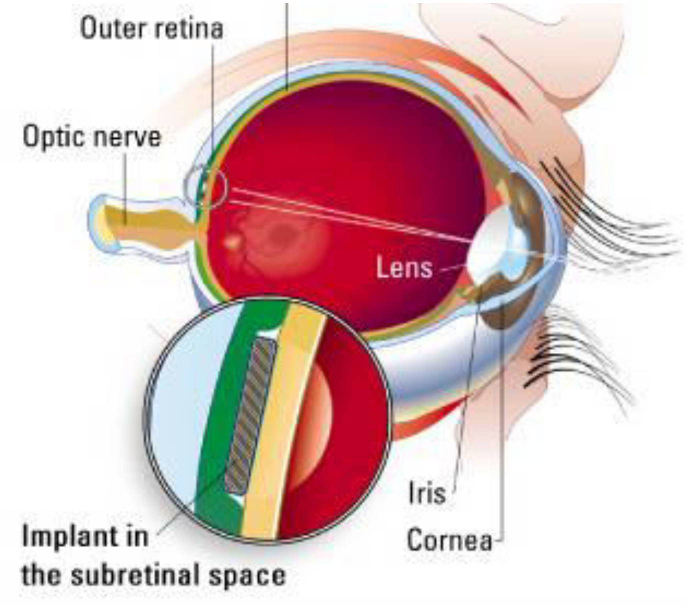
Larrissa Owens

Graduate Students, GK12 Fellow

Department of Mathematics, UTA

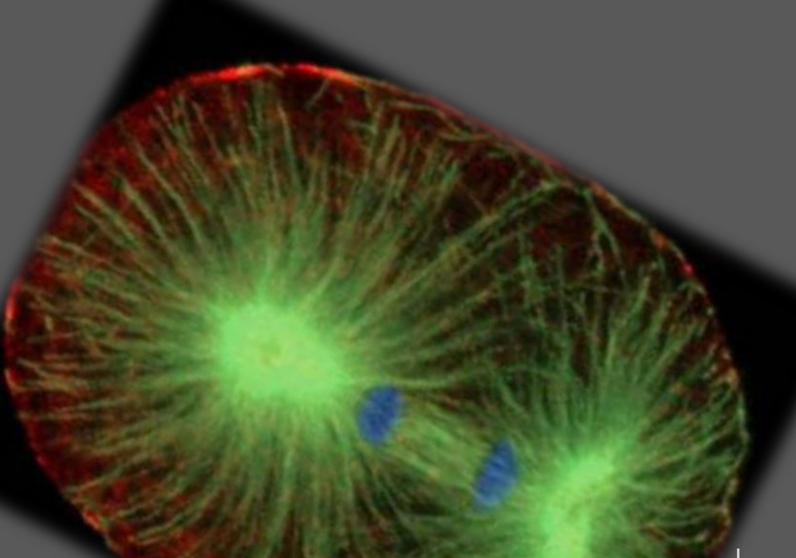


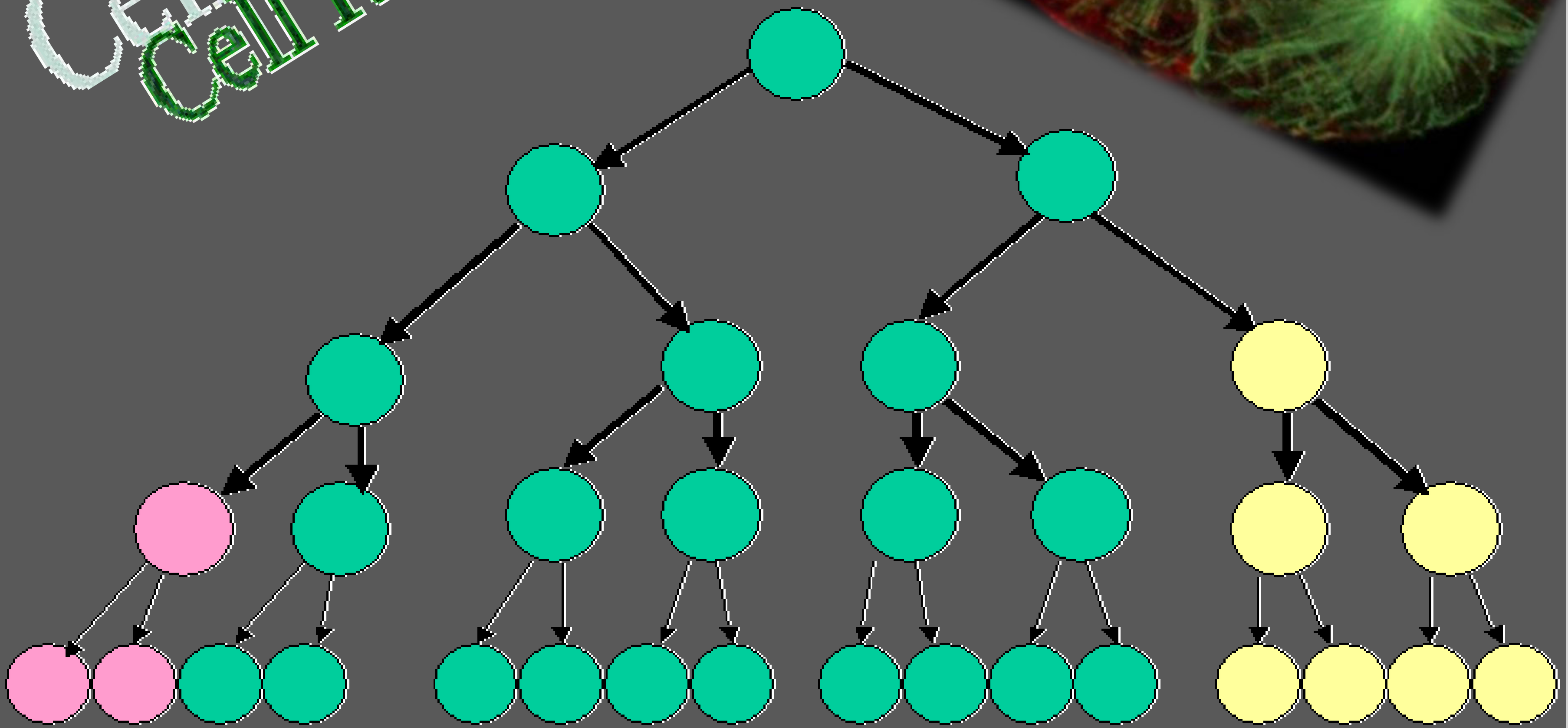
SIGHTS SET ON ADVANCING TREATMENT FOR RETINAL DISEASE



Cell Replication

Cell Replication





If you were to graph the Cell Replication what do you think it would look like?

If it replicates once each hour then you have:

Hour (t)	# of cells (y)
0	1
1	2
2	4
3	8
4	16

$$y = (?)t+1$$

What is the y-intercept?

Is there a constant value that you are multiplying the hour (independent variable) by where adding one to the result would give you the corresponding number of cells (the dependent variable)?

If you were to graph the Cell Replication what do you think it would look like?

If it replicates once each hour then you have:

Hour (t)	# of cells (y)
0	1
1	2
2	4
3	8
4	16

On your communicator, give a quick sketch of what you think the graph of the # of cells will look like over time.

If you were to graph the Cell Replication what do you think it would look like?

If it replicates once each hour then you have:

Hour (t)	# of cells (y)
0	1
1	2
2	4
3	8
4	16

Is there a constant value that you are multiplying the hour (independent variable) by where adding one to the result would give you the corresponding number of cells (the dependent variable)?

Linear Equation: $y = (?)t+1$

What do you think the equation of this graph might involve? Do you see any pattern in the column of cell amounts?

Hour (t)	# of cells (y)
0	1
1	2
2	4
3	8
4	16

What do you think the equation of this graph might involve? Do you see any pattern in the column of cell amounts?

Hour (t)	# of cells (y)
0	$1 = 2^0$
1	$2 = 2^1$
2	$4 = 2^2$
3	$8 = 2^3$
4	$16 = 2^4$

$$y = a\,b^t$$

What is the value of “a” for our model of cell replication ?

What is the value of “b” for our model of cell replication?

What do you think the equation of this graph might involve? Do you see any pattern in the column of cell amounts?

Hour (t)	# of cells (y)
0	$1 = 2^0$
1	$2 = 2^1$
2	$4 = 2^2$
3	$8 = 2^3$
4	$16 = 2^4$

$$y = 1 \times 2^t$$

What is the value of “a” for our model of cell replication ?

What is the value of “b” for our model of cell replication?

$$y = a\,b^{t}$$

What is the “t” referred to as in this equation?

$$y = a b^t$$

What is the "t" referred to as in this equation?

exponent

$$y = a b^t$$

What is the “t” referred to as in this equation?

What do you think they named equations of this form as a result?

exponent

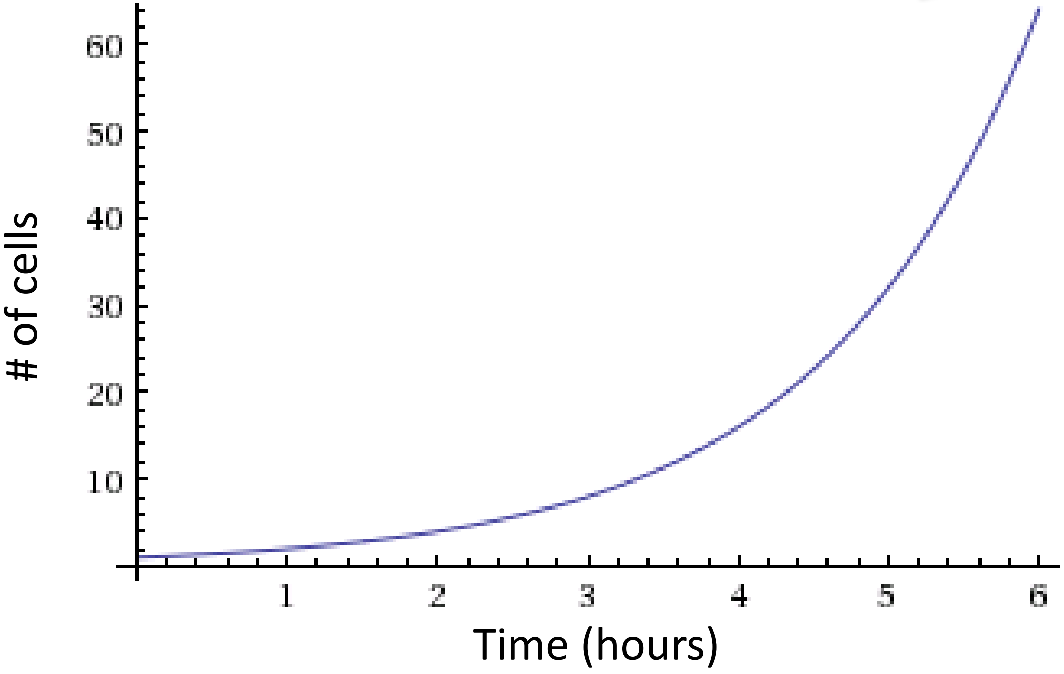
$$y = a b^t$$

What is the “t” referred to as in this equation? → What do you think they named equations of this form as a result?

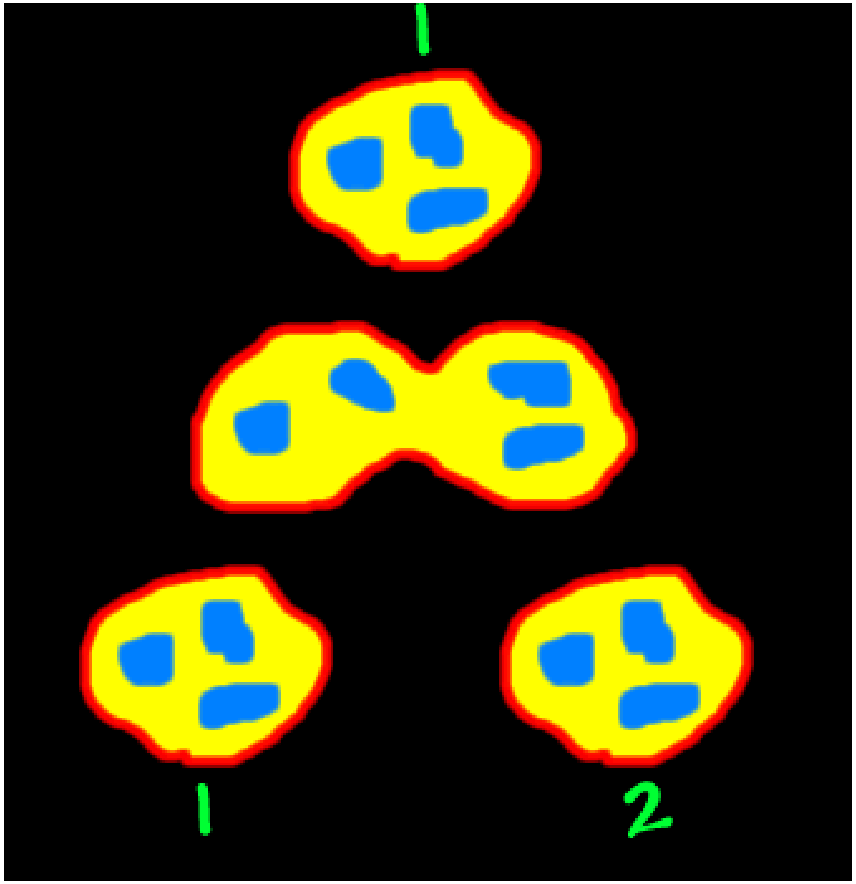
exponent → **exponential**

Exponential Growth

$y = 2^t$



Pop Quiz



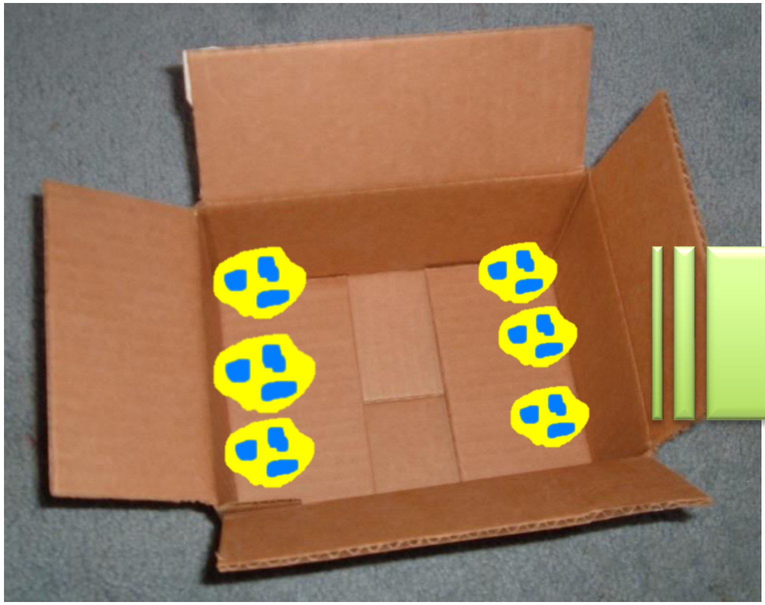
Assume: every minute a cells splits to form 2 new cells.

At 5pm you put one cell into a box.

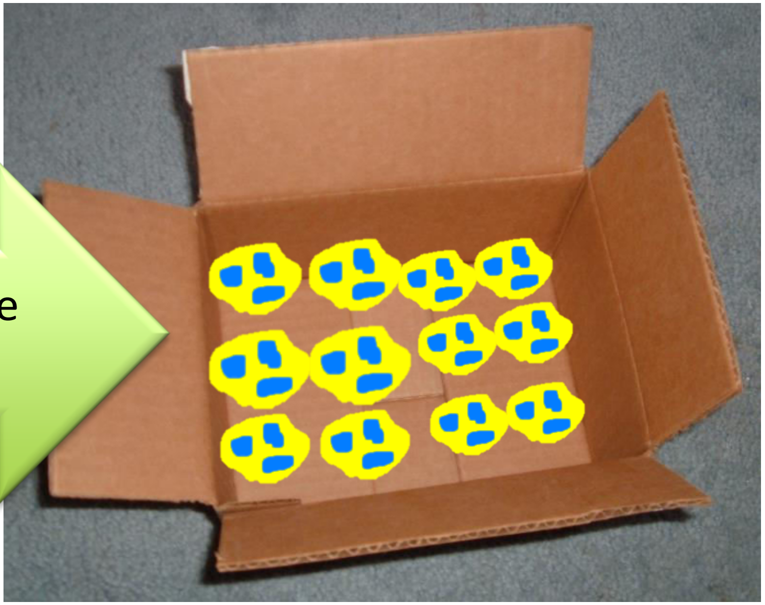
At 6pm the box first reaches full capacity.

When was the box half full?

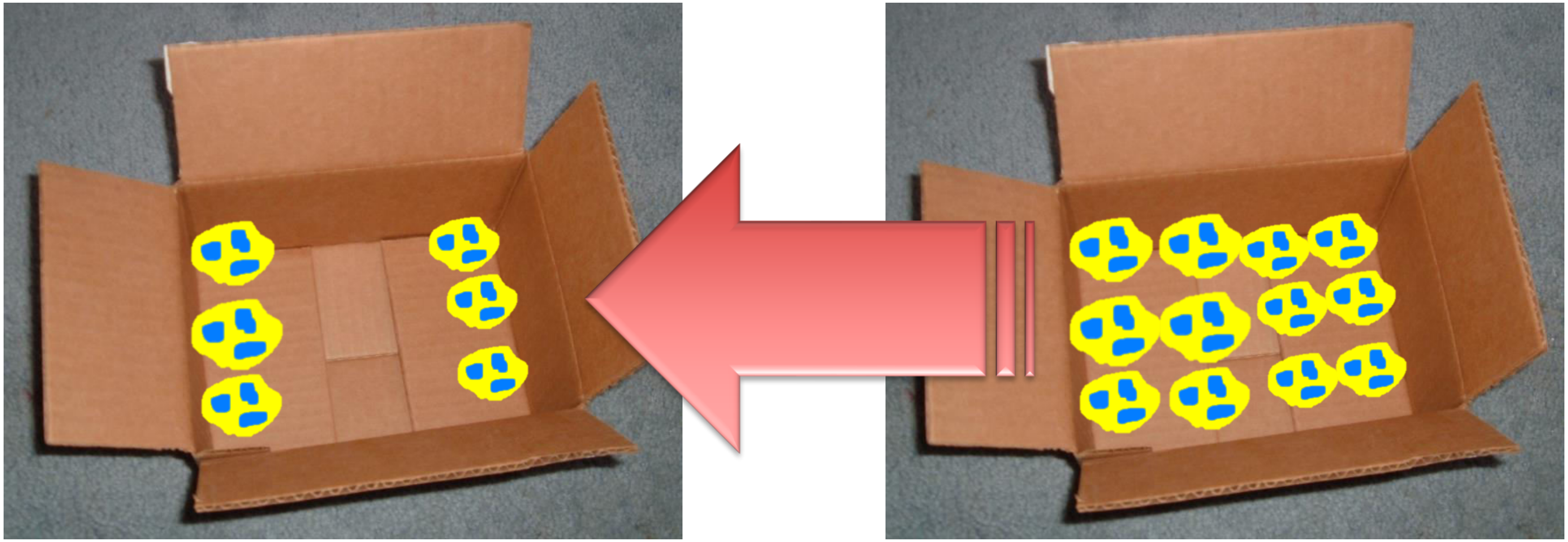
5:59



One Minute Later

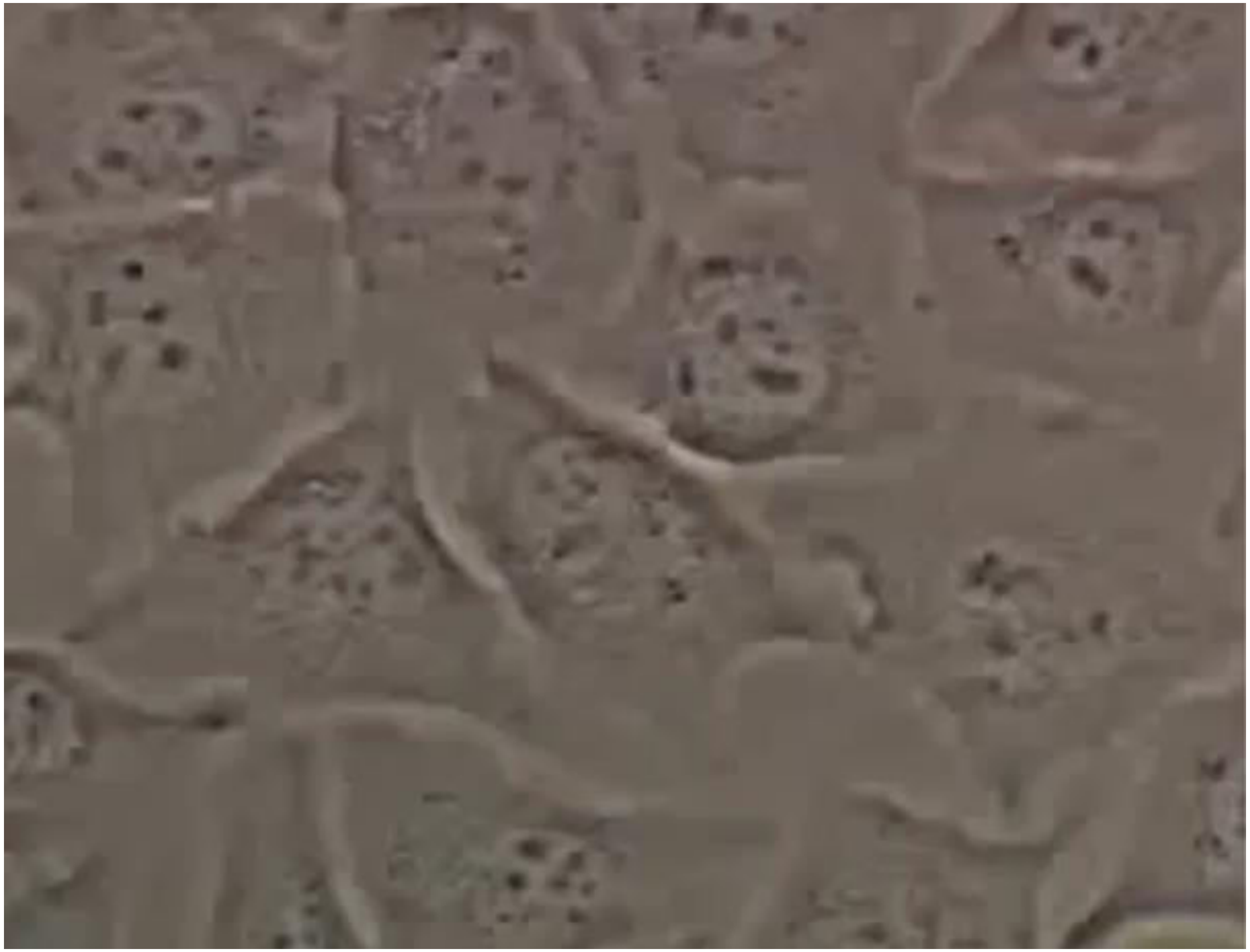


Exponential Decay



Recall

Apoptosis: Programed cell death



Half Life: Penny activity

What do you think the equation of this graph might involve? Do you see any pattern in the column of cell amounts?

Hour (t)	# of cells (y)
0	16
1	8
2	4
3	2
4	1

$$y = a\,b^{t}$$

What is the value of “a” for our model of cell replication ?

What is the value of “b” for our model of cell replication?

What do you think the equation of this graph might involve? Do you see any pattern in the column of cell amounts?

Hour (t)	# of cells (y)	
0	16 = 16	$=16 \times .5^0$
1	8 = 16 x .5	$=16 \times .5^1$
2	4 = 16 x .5 x .5	$=16 \times .5^2$
3	2 = 16 x .5 x .5 x .5	$=16 \times .5^3$
4	1 = 16 x .5 x .5 x .5 x .5	$=16 \times .5^4$

$$y = 16\,(0.5)^t$$

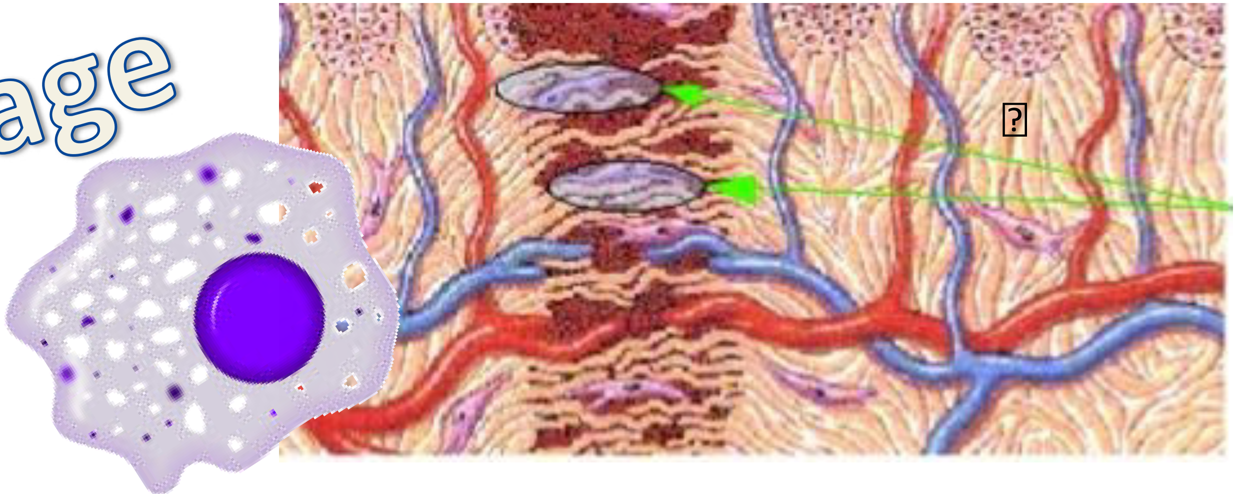
We start with 16

We divide our populationin half after each hour?

Clearing out your calculator:

- Press **Y=** and hit **CLEAR**
- Press **2nd** and **+** then press **4** and **enter** . The screen Should say “Clr All Lists” then **Enter** one more time

Macrophage



Isolating just the decay term of the macrophages:

$$\frac{dD}{dt} = -f_0\lambda_1 MD + f_0\lambda_3 M$$

$$\frac{dC}{dt} = f_1 D + f_2\lambda_3 M - f_3\lambda_2 MC - f_4 C$$

$$\frac{dF}{dt} = a_{12}FC + a_2 F\left(1 - \frac{F}{F_0}\right) - a_3 F$$

$$\boxed{\frac{dM}{dt} = a_{11}MCH(M_0 - M) - a_0 M}$$

$$\frac{dE}{dt} = a_{16}F\left(1 - \frac{E}{E_0}\right)$$

$$\frac{dM}{dt} = a_{11}MCH(M_0 - M)\boxed{- a_0 M}$$

$$-a_0 M$$

Isolating just the decay term of the macrophages, we get the equation

$$M=(\text{initial amount}) \times e^{-0.045t}$$

e =2.718281828459045235360287471352...

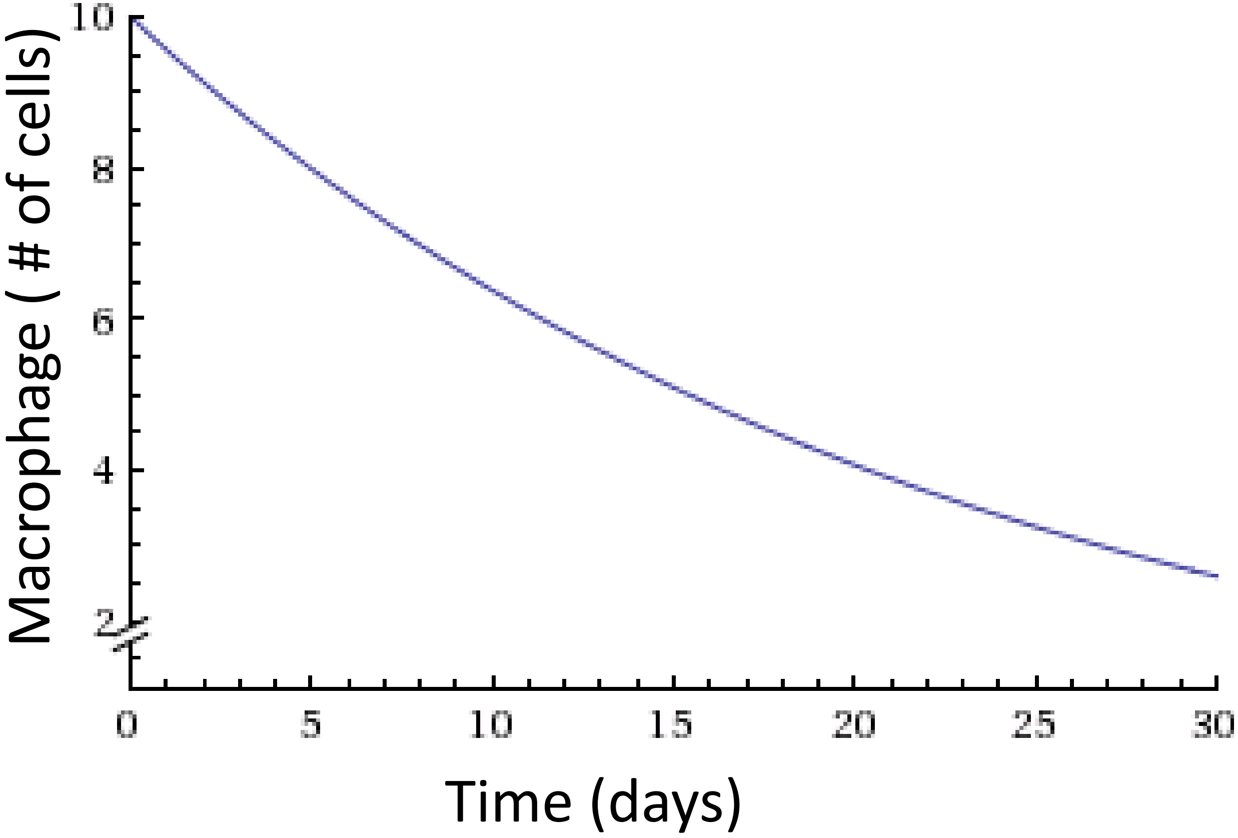
Quick Review: What type of number is e, rational or irrational?

If we start with 10 macrophages then our equation for modeling the growth is:

$$M=10 \times (2.718)^{-0.045t}$$

If we start with 10 macrophages then our equation for modeling the growth is:

$$M=10 \times (2.718)^{-0.045t}$$



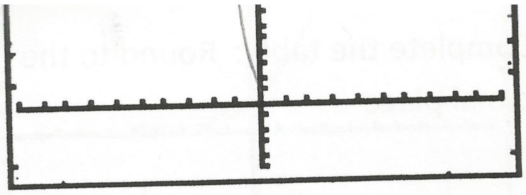
Lesson 6

Reflections:
Over all this lesson really had the students engaged and encountering new trend that challenged the linear behaviors they were so used to observing. The students answered my questions just as I had anticipated, they first assumed that there would be a way to form a linear equation to accurately produce the table they were given, as soon as we allowed the students to draw the points they could easily see that the plots were non-linear and therefore different from the tables they are used to finding equations for. By establishing the pattern in the table the students were able to easily follow the reasoning behind why an exponent is needed and began to grasp the idea behind the symbolic structure of an exponential function. This was good, but to test whether or not they where understanding beyond the symbolic structure they were asked a pop quiz question which quickly helped them realize that they were still assuming linear qualities about the behavior of the exponential scenario which led them to drawing incorrect conclusions. Providing a visual of the exponential behavior, showing how many cells are previous at one time step and then the next they started to understand where their reasoning was flawed. Working with the pennies as a physical prop to represent cell death really helped the remaining students grasp the concept. It was cool to watch as each student really began to grasp the concept at different times in the lesson. Through observing the time constraint in the first class I gave a little more guidance in the activity portion of the lesson to insure that students would finish in time and see how calculators can help us relate our experiment to mathematical models. Students responded well to both the communicator interaction, the pop quiz and the penny activity. Of all of the activities I think the pop quiz was really the most common " light bulb moment" for them. Their questions and curiosity about the cell replication and the behavior of the growth and decay confirmed that they were engaged and able to observe how the mathematics related to the real world behaviors. Teaching this lesson really showed me the importance of multiple representations as well as different learning styles because some students were really able to understand it from the symbolic representation, others gleaned the most from the verbal discussion during the pop quiz, while others understood the behavior better after experimenting with it with objects they could touch. I don't know that I really have anything that I would change other than to go back and wish 1st period had received more instruction so that they would have more time to complete the experiment.

For the extension assignment in the future I would include a function along the lines of y=(4/3)^x so that the students would have a non-whole number increasing example in their tool box as they worked to discern qualities that determine whether an exponential function is increasing or decreasing.

(circle one)

X	Y
-2	100
-1	10
0	1
1	.1
2	.01



8. What point do they all have in common? (0, 1)

9. List the equations that are increasing. $y=2^x$, $y=3^x$, $y=10^x$, $y=1^x$

10. List the equations that are decreasing. $y=(1/3)^x$, $y=(1/10)^x$, $y=(1/2)^x$,

* 4 good observation, but generalization presents a problem if extended beyond this collection to include improper fractions

11. When an exponential graph is increasing , it shows expo
What are the similarities of all the equations that produce

They are all whole numbers. larger than one.

* I caught this student who originally just wrote "whole numbers" and had them try $y=\left(\frac{5}{3}\right)^x$ at which point they lengthened their answer

ations that are decreasing. $y=(1/10)^x$, $y=(1/3)^x$, $y=(1/2)^x$

an exponential graph is increasing , it shows exponential growth .
What are the similarities of all the equations that produce such graphs?

All whole numbers or improper fraction

bigger than 1= increaser
Smaller than 1= decreasing

Great additional observations about the behavior of the graphs!

11. When an exponential graph is increasing , it shows exponenti
What are the similarities of all the equations that produce suc

the increasing is a mirrior of the decreasing equa
it its bigger than one it increases and if it is smaller it
decreasing