NSF GK-12 MAVS Project Lesson Write Up Template

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Title of Lesson: Latin Squares

Class: 8th grade math

Topic: problem solving, characteristics of Latin squares

Objectives:

TLW be able to identify a Latin Square, justify why a set of numbers makes a Latin Square and create their own Latin Squares using manipulatives, colors, and numbers.

Standards TEKS:

- 8.15(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models;
- 8.16(A) make conjectures from patterns or sets of examples and non-examples;

Standards NCTM:
Problem Solving

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Key vocabulary: Latin square

List Materials and Resources used: Fellow created worksheet, projector for presentation, sets of colored tiles (4 red, 4 blue, 4 yellow, 4 green), as well as a colored pencil of each color, for each student

Research Setting/Connection/Motivation: (This should be a layperson's description of the connections to research in mathematics. We should strive to make this a short paragraph.)

The multiplication tables for semifields are Latin squares. Due to this, one method of finding semifields is to examine appropriate Latin squares. This lesson is an introduction to the concept of Latin squares and how much information is required o obtain a unique solution to a Latin square.

Prior knowledge: Provide a short description of what students have already learned in the course that is relevant for this lesson

None

Vertical Strands:
NCTM: Problem Solving

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts

- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

TEKS:

7th grade:

- 7.13 (D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
- 7.15(A) make conjectures from patterns or sets of examples and nonexamples; and 7.15(B) validate his/her conclusions using mathematical properties and relationships.

8th grade:

- 8.15(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models;
- 8.16(A) make conjectures from patterns or sets of examples and non-examples;

Algebra 1, Geometry, Algebra 2, Math Models, Pre-Calculus:

Basic Understanding:

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts. :

Latin Squares

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A Special Kind of Latin Square

	3				1		7	
6			8					2
		1		4		5		
	7				2		4	
2				9				6
	4		3				1	
		5		3		4		
1					6			5
	2		1				3	

Definition of a Latin Square

A Latin square of size n is a square n-by-n grid whose entries all belong to the same set of n symbols with no symbol appearing in the same row or column more than once.

If the set of symbols is 1, 2, and 3, then the following is a Latin square of size 3:

1	2	3
3	1	2
2	3	1

And the following is NOT a Latin square:

1	2	3
1	2	3
1	2	3

REMEMBER!

A 3-by-3 Latin square will only have three colors in it!

Operations Can Create Latin Squares

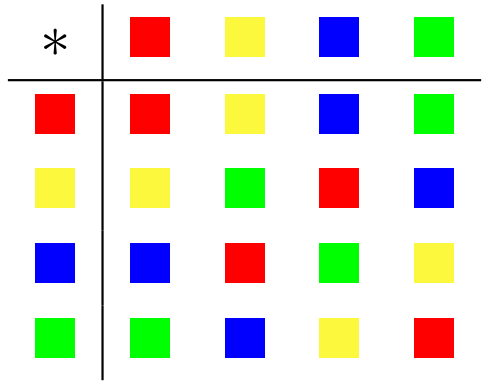
Look at the addition and multiplication tables for $\mathbb{Z}_5$. The inside are Latin squares.

$+$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

$*$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

Latin Squares Define Operations

We can define a new operation using a Latin square. Using 3.B. we can define a multiplication of colors.



Notice that multiplication by red does not change a color. In professional mathematics we would call the colors **elements** and red would be the **identity element**.

My Research - Semifields

The mutliplication table of a semifield is a Latin square which satisfies certain conditions, including the existence of an identity element. So, to discover all of the semifields of a particular size, we can look at all of the Latin squares of that size.

Name_____ Date_____

Latin Squares

1.Complete the following Latin squares, using the symbols listed below the squares.

1	3	2
3	2	1
2	1	3

1, 2, 3

Δ	O	Π
Π	Δ	O
O	Π	Δ

Δ, O, Π

B	A	C
A	C	B
C	B	A

A, B, C

2. Use the colored tiles to create your own 3-by-3 Latin square using only 3 colors. Color the following square when you are done.

R	G	B
B	R	G
G	B	R

3. Use the colored tiles to complete the following Latin squares. The letters refer to which color each square should be, R = red, G = green, B = blue, and Y = yellow.

A.

G	B	Y	R
Y	R	B	G
B	G	R	Y
R	Y	G	B

B.

R	Y	B	G
Y	G	R	B
B	R	G	Y
G	B	Y	R

C.

G	R	B	Y
R	Y	G	B
B	G	Y	R
Y	B	R	G

4. Use the colored tiles to complete the following Latin squares. There is more than one solution for each square.

A.

R	B	Y	G
Y	G	R	B
G	Y	B	R
B	R	G	Y

B.

Y	B	R	G
G	Y	B	R
R	G	Y	B
B	R	G	Y

C.

B	R	G	Y
R	B	Y	G
Y	G	R	B
G	Y	B	R

5. Now use the colored squares to create a 4-by-4 Latin square using 4 colors. Color the following square when you are done.

R	G	B	Y
Y	R	G	B
B	Y	R	G
G	B	Y	R

6. Describe, in words, how you came up with your Latin square. Did you just try random combinations, or did you come up with some sort of method?

I started with the row "RGBY", and shifted each letter right by 1 for each new row.

7. Which of the following are Latin squares? Explain why or why not.

A.

0	1	2	3	4
1	2	3	4	0
2	3	4	0	1
3	4	0	1	2
4	0	1	2	3

B.

0	0	0	0	0
0	1	2	3	4
0	2	4	1	3
0	3	1	4	2
0	4	3	2	1

C.

1	2	3	4
2	4	1	3
3	1	4	2
4	3	2	1

B is the only square which is not a Latin square, because it has 0 in multiple rows and columns.

Name_____ Date_____

Latin Squares

1.Complete the following Latin squares, using the symbols listed below the squares.

1		
	2	
		3

1, 2, 3

Δ		
		O
O		

Δ, O, Π

		B
	B	A

A, B, C

2. Use the colored tiles to create your own 3-by-3 Latin square using only 3 colors. Color the following square when you are done.

3. Use the colored tiles to complete the following Latin squares. The letters refer to which color each square should be, R = red, G = green, B = blue, and Y = yellow.

A.

G			
Y	R		G
B		R	Y
			B

B.

	Y		G
Y		R	
	R		Y
G		Y	

C.

G	R		
	Y		B
B		Y	
		R	G

4. Use the colored tiles to complete the following Latin squares. There is more than one solution for each square.

A.

R			
	G		
		B	
			Y

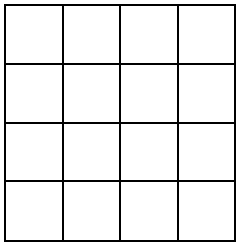
B.

Y			
	Y		
		Y	
			Y

C.

	R		
R			
			B
		B	

5. Now use the colored squares to create a 4-by-4 Latin square using 4 colors. Color the following square when you are done.



6. Describe, in words, how you came up with your Latin square. Did you just try random combinations, or did you come up with some sort of method?

7. Which of the following are Latin squares? Explain why or why not.

A.

0	1	2	3	4
1	2	3	4	0
2	3	4	0	1
3	4	0	1	2
4	0	1	2	3

B.

0	0	0	0	0
0	1	2	3	4
0	2	4	1	3
0	3	1	4	2
0	4	3	2	1

C.

1	2	3	4
2	4	1	3
3	1	4	2
4	3	2	1

Lesson Presentation

1. Engagement:
   The students will be presented with a Sudoku puzzle and asked if they know what it is. If so, they will be asked to explain the rules for solving a Sudoku. This will lead to the definition of a Latin square, which a Sudoku s a special case of.

2. Exploration:
   The students will be asked to complete a set of 3-by-3 Latin squares, and then construct their own 3-by-3 Latin square.

3. Explanation:
   The students will complete 4-by-4 Latin squares which have unique solutions, and then they will complete 4-by-4 Latin squares which have multiple possible solutions. The key difference is the number of completed squares provided and their position.

4. Elaboration:
   The students will be asked to create their own 4-by-4 Latin square. They will then be asked to explain any methods they used to create this square.

5. Evaluation:
   The students will complete the worksheet. The final problem tasks them with determining which squares in a set are Latin squares. The provided squares are actually the interiors of addition and multiplications tables for modular addition and multiplication. This will lead to a discussion of how certain mathematical operations create Latin squares, and how Latin squares can be used to define operations.

Lesson Reflection

1. What went well?
   Everything. This activity was my most successful so far, and will probably be the most successful overall. The students were very engaged and seemed to both enjoy the work and be challenged by it.

2. What didn't go well?
   Some students finished really quickly and there wasn't anything else for them to do.

3. What would you keep?
   I would keep everything as it is.

4. What would you change?
   Nothing.

Teacher Notes

This activity seems like it could be used in a wide range of classes. It assumes little prior knowledge, and focuses on logic, problem solving, and critical thinking in a universal way. Each student was given a bag containing red, green, yellow, and blue colored pencils, along with 4 square tiles in each of these colors (16 tiles total). The students were not required to use these to complete the worksheet. Some preferred to simply use the letters R, G, Y, and B, and never even opened their bags. Other students were completely unable to solve the squares without using the tiles. I feel that this flexibility was part of what made this activity so successful.

Problems 2, 4, 5, and 6 on the worksheet do not have unique solutions. On the answer key, example solutions are given, but it is likely that students will provide different answers. When checking a Latin square, simply use the rule that no number can appear in the same row or column more than once.

The included Sudoku worksheet can be given to students who are particularly good at working with Latin squares and finish the worksheet quickly.

Name_____ Date_____

Sudoku Puzzles

In a Sudoku puzzle, the numbers 1 through 9 appear only once in each row, column, AND bordered box.

1. Here are Sudoku puzzles of different difficulties. If you were not told, how could you tell which puzzles were the easiest or hardest?

Very Easy

6			1		8	2		3
	2			4			9	
8		3			5	4		
5		4	6		7			9
	3						5	
7			8		3	1		2
		1	7			9		6
	8			3			2	
3		2	9		4			5

Easy

	7	8			4		3	1
		4	9	5				8
3							6	
				7	6			9
			4		9			
9			5	1				
	1							3
6				4	7	5		
2	9		6			8	1	

Medium

5	4		2		9			1
			5					4
		7				9		
8				3			6	7
			6		5			
9	3			1				2
		1				6		
2					6			
3			1		7		4	9

Hard

			8	4				9
		1						5
8				2	1	4	6	
7		8					9	
	5					3		1
	2	4	9	1				7
9						5		
3				8	4			

Very Hard

	2							7
	7				4		1	
9		5						
	8		6	3				2
7								1
2				1	8		6	
						4		9
	3		1				2	
4							8	

"World's Hardest" 6-27-2012

8								
		3	6					
	7			9		2		
	5				7			
				4	5	7		
			1				3	
		1					6	8
		8	5				1	
	9					4		