

NSF GK-12 MAVS Project Lesson Plan Template

Lesson 6: RELIABILITY OF BREAKFAST TIMES

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Class: Pre-AP Precalculus

Topic: Statistical measure of reliability

Objectives: Curriculum objectives: Students will collect, analyze, summarize and display data of their own breakfast times recorded on school days. Students will compare their results with the class and interpret similarities and differences. Research objectives: Students will explore the connection between the reliability of their breakfast times and the reliability of burst times generated by a noisy mathematical model of neuron bursts.

Standards: **TEKS-(3)** The student uses functions and their properties, tools and technology, to model and solve meaningful problems. The student is expected to:

(C) use regression to determine the appropriateness of a linear function to model real-life data (including using technology to determine the correlation coefficient);

NCTM-• for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics;

Key vocabulary: mean, median, standard deviation, 5-number summary, coefficient of variation, raster plot, histogram, reliability

Materials and Resources: giant sticky-note graph paper (2 sheets, plus an additional 2 sheets for each group of 3 or 4), 2 individual activity sheets for each student, 1 group activity sheet for each group of 3 or 4, graphing calculators (TI-83 or 84), document camera, powerpoint intro slides

Research Setting/Connection/Motivation: Mathematically noisy models of neuron bursts are those in which a Gaussian “white noise” term is included in one of the differential equations. Multiple trials are run for a given level of “noise” in order to evaluate the reliability of burst times generated by the model. These simulations can be repeated for other given noise levels as well. A measure of the model’s reliability in terms of burst timing is calculated by first determining the optimal “bin width” of a histogram of burst times. Students can adapt this approach to determine their own reliability measure of the times at which they eat breakfast on school days.

Prior knowledge: Students should have an understanding of basic statistics, including formulas which include summation notation.

Lesson Presentation:

PREP ACTIVITIES (Engage & Explain)

1. Students should record their breakfast times for 21 school days prior to the lesson.
 - a. First page of document, “BREAKFAST LOG,” is an assignment of each student to a code name, which automatically puts them in a group of 3 or 4. This will be useful for later group activities.
 - b. The remaining pages of “BREAKFAST LOG” provide a convenient way for students to record their breakfast times as they go, via code names. These logs can be kept in the classroom and students can update as desired.
2. Mini-lesson on the day prior to the lesson on summation notation and calculation of mean and variance (Explore & Explain, 15 minutes)

- a. Display the “STEP THREE Histograms and Reliability” activity sheet on the document camera, focusing on item 1. a., ii and iii.
- b. As an example, let $k_1 = 3$, $k_2 = 12$, $k_3 = 5$, $k_4 = 1$, and $k_5 = 0$
- c. Prompt students and/or guide them to calculate mean and variance via summation notation.

INTRO – RESEARCH SLIDES: “Intro slides 6,” 1 – 10 (Engage, 10 min)

STEP ONE ACTIVITY – “STEP ONE Indiv Stats & Raster” (Explore, Explain & Evaluate, 25 min)

1. Prompt students to convert times to decimals.
2. Model with graphing calculator (via document camera) how to enter a list. (Explain)
3. Calculate individual stats: mean, SD, 5-number summary, CV (Coefficient of Variation). Model with graphing calculator how to generate 1-var stats. (Explore)
4. Create individual raster plot; model on doc camera and facilitate discussion: (Explain & Explore)
 - a. Each student will have to determine an appropriate range of values for their breakfast times and then scale & label their horizontal axis accordingly.
 - b. What do you learn from this plot?
 - c. Compare your plot to a couple of other people. What’s different?
5. Compile large class graph of CVs (use sticky dots). (Evaluate)
 - a. Determine range of values for CV, and create the large graph (x-axis will list code name for each student; determine y-axis upon obtaining overall min CV and overall max CV of all students). The range of CV values should be between 0 and about 0.4.
 - b. What does CV tell us about a data set?
 - c. What is this graph telling us?

STEP TWO ACTIVITY – “STEP TWO Small Group 5-number Summary” (any part may be eliminated for time) (Extend & Evaluate, 20 min)

1. Each group of 3 or 4 will be given giant sticky-note graph paper. Each student within a group will create box-plots of their 5-number summary from STEP ONE on the same graph. (Extend)
 - a. Use the x-axis is for code names of each student (e.g. T1, T2 and T3).
 - b. The y-axis is numeric (i.e., 0 to number large enough to contain highest value of every student in group).
2. Compare & Contrast questions (Evaluate)
3. Each group create raster plot on giant sticky note. Write code names on individual students’ sticky dots. (Extend)

STEP THREE ACTIVITY – “STEP THREE Histograms and Reliability” (Explain, Explore & Evaluate, 30 min)

1. Each student will construct a relative frequency histogram (the period T , in minutes, will be determined by instructor based on all of student data). To begin, let $N = 5$. (Explain, Extend & Evaluate)
 - a. Were you surprised by your optimal bin width?
 - b. Compare histograms.
 - c. Would you scale the vertical axis any differently? Explain.
2. Each student will calculate overall individual “reliability” (Explore)
3. Compile class graph of “reliability” measure with giant sticky-note graph paper and sticky dots. Once again, the horizontal axis will be student code names. The vertical axis ranges from 0 to 1 . (Evaluate)
 - a. Whose breakfast times are most reliable?
 - b. What would you expect to happen to reliability if we included lunch and supper times?

WRAP-UP: CONNECTION TO RESEARCH (Explain & Extend, 5 min)

“Intro slides 6,” slides 11 – 14

Vertical Strands:

MMA - M.2A interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, line plots, stem and leaf plots, and box and whisker plots to draw conclusions from the data

MATH 1308 Elementary Statistical Analysis: Students will be able to construct histograms and compute probabilities using histograms; Students will be able to compute the average, standard deviation, median and identify outliers of a set of data. They will also be able to interpret these statistics and make use of these statistics in analyzing data.

MATH 3316 Statistical Inference: Students will be able to use a computer package to analyze data. They will be able to compute and interpret univariate and bivariate descriptive statistics and implement these methods for the purposes of exploring a large data set towards addressing specific questions.

MATH 5305 Statistical Methods: Students will collect, analyze and summarize univariate data with various measures of position and dispersion.

Research Fellow Notes & Reflections:

Before addressing each point of the lesson, it is important to note that since this lesson focuses on statistics, students in Pre-AP Precalculus generally do not address such topics during their course. Some students remembered some things about statistical summaries and graphical representations from a previous course, but even those were sketchy memories. If I had it to do over, and the curriculum allowed time for it, I would have created about 3 mini-lessons about statistics in advance of this research lesson. Mrs. Geppert, mentor teacher, affirmed that this lesson was well worth it for those students who had expressed interest in enrolling in AP Statistics for their senior year. So the lesson stimulated interest from that perspective.

PREP:

The breakfast logs were quite helpful to students who were concerned about losing or forgetting their recorded breakfast times. We had to discuss the definition of “eating breakfast” for some. So we decided that one’s breakfast would be the first time of the day when one ingested something, even if just a drink of water. The reason for this was that some students claimed they did not eat breakfast, and/or may not eat until evening hours. This facilitated a discussion of “outliers” in terms of statistical data during the data collection process. As a courtesy, for those students who used the log, I entered their data on a spreadsheet, printed it out and cut individual strips for each student the day before the lesson.

During the mini-lesson on summation notation, I discovered that the students were somewhat familiar with the notation, but appreciated the review & practice. However, they were not familiar with statistical variance. So the lesson took a little longer than 15 minutes, maybe on the order of 25 minutes. On a positive note, I was able to use this mini-lesson opportunity to explain what a histogram was (they were vaguely familiar with that), and how we would use it to display breakfast times.

INTRO – RESEARCH SLIDES:

I took too much time on this, 5 – 7 minutes longer than I should have. Slide 8 should be simplified or eliminated from this particular lesson.

STEP ONE ACTIVITY:

Wake-up call! This step took most of the rest of our class time, which is a 90-minute class. Thankfully, Mrs. Geppert was happy to let me take two full class days to do this entire research lesson. Here are some notes and recommendations:

1. Converting times to decimals took up too much class time. I recommend that students record as both times *and* decimals during the 21 days of data collection (both representations will be needed).
2. When I realized it would take most of the class time to complete the STEP ONE ACTIVITY, *and* that the STEP THREE ACTIVITY was the most important research connection that I wanted to make, I chose to skip the STEP TWO ACTIVITY (see exception below).
3. Some students completed their individual raster plots before the end of the class. So I had those students prepare a group raster plot (on giant sticky-note graph paper) in order to compare with each other. This was one of the STEP TWO ACTIVITY items.

STEP THREE ACTIVITY:

This activity began the second day of the lesson. Only a handful of students completed this entire activity. When I realized that not all would be able to complete it, I instructed most students to skip part 1. b. The reason for this is that the measure of reliability statistic was more important to my research, and allowed for easy comparison of reliability among the students' breakfast times.

Note about part 1. a. iii : Encourage students to use the list and stat features of the graphing calculator, as opposed to calculating by hand the mean and variance.

Note about part 1. a. iv : A small handful of students had outliers because I chose a period of $T = 300$ minutes. For instance, if a student had say one outlier, their cost function would have a "20" in the denominator instead of "21".

About half of the students were able to complete part 2. by placing a sticky dot on the giant sticky-note graph for their reliability measure.

WRAP-UP:

I made sure to allow for 5 minutes to wrap-up and connect to my research. However, I kind of wish I had explained slide 11 at the end of the first day of the lesson.

NAME	NUMBER
	H-1
	H-2
	H-3
	O-1
	O-2
	O-3
	U-1
	U-2
	U-3
	S-1
	S-2
	S-3
	S-4
	T-1
	T-2
	T-3
	N-1
	N-2
	N-3
	Z-1
	Z-2
	Z-3
	Z-4

NUMBER	TUES-A 4/3/12₁	WED-B 4/4/12₂	THURS-A 4/5/12₃	MON-B 4/9/12₄	TUES-A 4/10/12₅	WED-B 4/11/12₆	THURS-A 4/12/12₇
H-1							
H-2							
H-3							
O-1							
O-2							
O-3							
U-1							
U-2							
U-3							
S-1							
S-2							
S-3							
S-4							

NUMBER	TUES-A 4/3/12₁	WED-B 4/4/12₂	THURS-A 4/5/12₃	MON-B 4/9/12₄	TUES-A 4/10/12₅	WED-B 4/11/12₆	THURS-A 4/12/12₇
T-1							
T-2							
T-3							
N-1							
N-2							
N-3							
Z-1							
Z-2							
Z-3							
Z-4							

PERIOD: Tuesday, APRIL 3RD – Thursday, APRIL 12TH

NUMBER	FRI-B 4/13/12 ₈	MON-A 4/16/12 ₉	TUES-B 4/17/12 ₁₀	WED-A 4/18/12 ₁₁	THURS-B 4/19/12 ₁₂	FRI-A 4/20/12 ₁₃	MON-B 4/23/12 ₁₄
H-1							
H-2							
H-3							
O-1							
O-2							
O-3							
U-1							
U-2							
U-3							
S-1							
S-2							
S-3							
S-4							

NUMBER	FRI-B 4/13/12₈	MON-A 4/16/12₉	TUES-B 4/17/12₁₀	WED-A 4/18/12₁₁	THURS-B 4/19/12₁₂	FRI-A 4/20/12₁₃	MON-B 4/23/12₁₄
T-1							
T-2							
T-3							
N-1							
N-2							
N-3							
Z-1							
Z-2							
Z-3							
Z-4							

PERIOD: Friday, APRIL 13TH – Monday, APRIL 23RD

NUMBER	TUES-A 4/24/12 ₁₅	WED-B 4/25/12 ₁₆	THURS-A 4/26/12 ₁₇	FRI-B 4/27/12 ₁₈	MON-A 4/30/12 ₁₉	TUES-B 5/1/12 ₂₀	WED-A 5/2/12 ₂₁
H-1							
H-2							
H-3							
O-1							
O-2							
O-3							
U-1							
U-2							
U-3							
S-1							
S-2							
S-3							
S-4							

NUMBER	TUES-A 4/24/12 ₁₅	WED-B 4/25/12 ₁₆	THURS-A 4/26/12 ₁₇	FRI-B 4/27/12 ₁₈	MON-A 4/30/12 ₁₉	TUES-B 5/1/12 ₂₀	WED-A 5/2/12 ₂₁
T-1							
T-2							
T-3							
N-1							
N-2							
N-3							
Z-1							
Z-2							
Z-3							
Z-4							

PERIOD: Tuesday, APRIL 24TH – Wednesday, MAY 2ND



Alice Lubbe, Math Student



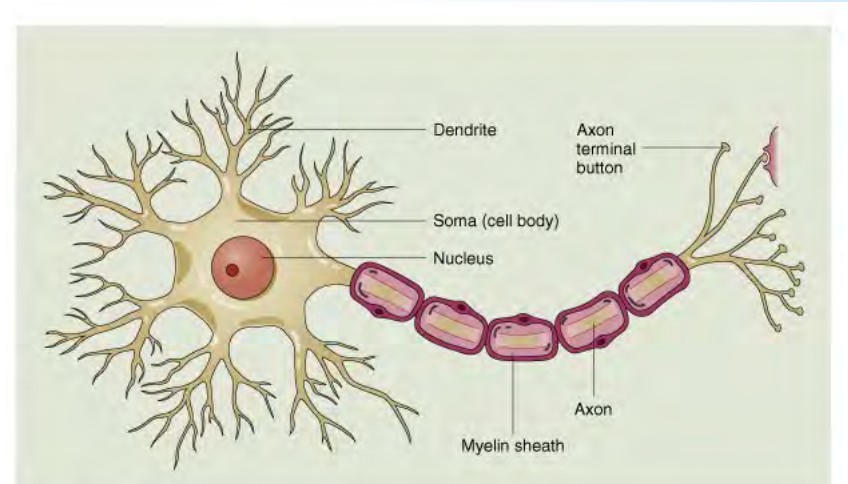
remember the bats →



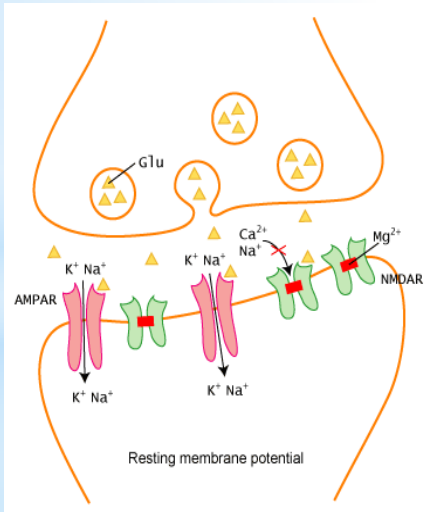
Noisy Neural Bursting Models and Associated Signal Reliability

Alice Lubbe (Ph.D. Candidate)
Supervising Professor: Jianzhong Su, Ph.D.

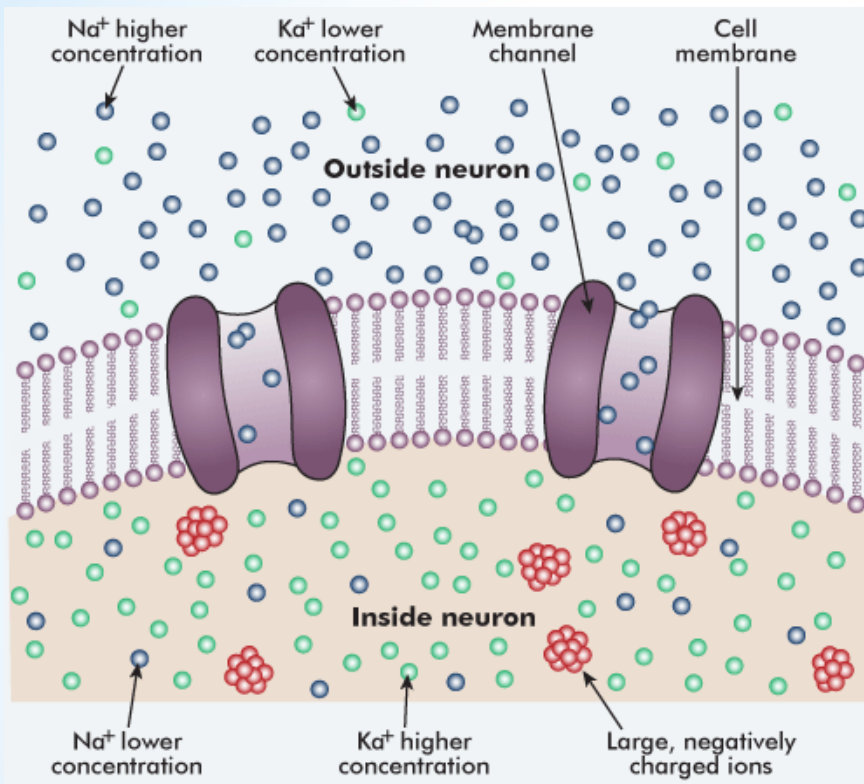
I study systems of math equations which describe changes in a neuron's behavior.



Neurons are the building blocks of the nervous system. Electrical signals transmit information from the cell's nucleus, through the axon, to axon terminals . . .

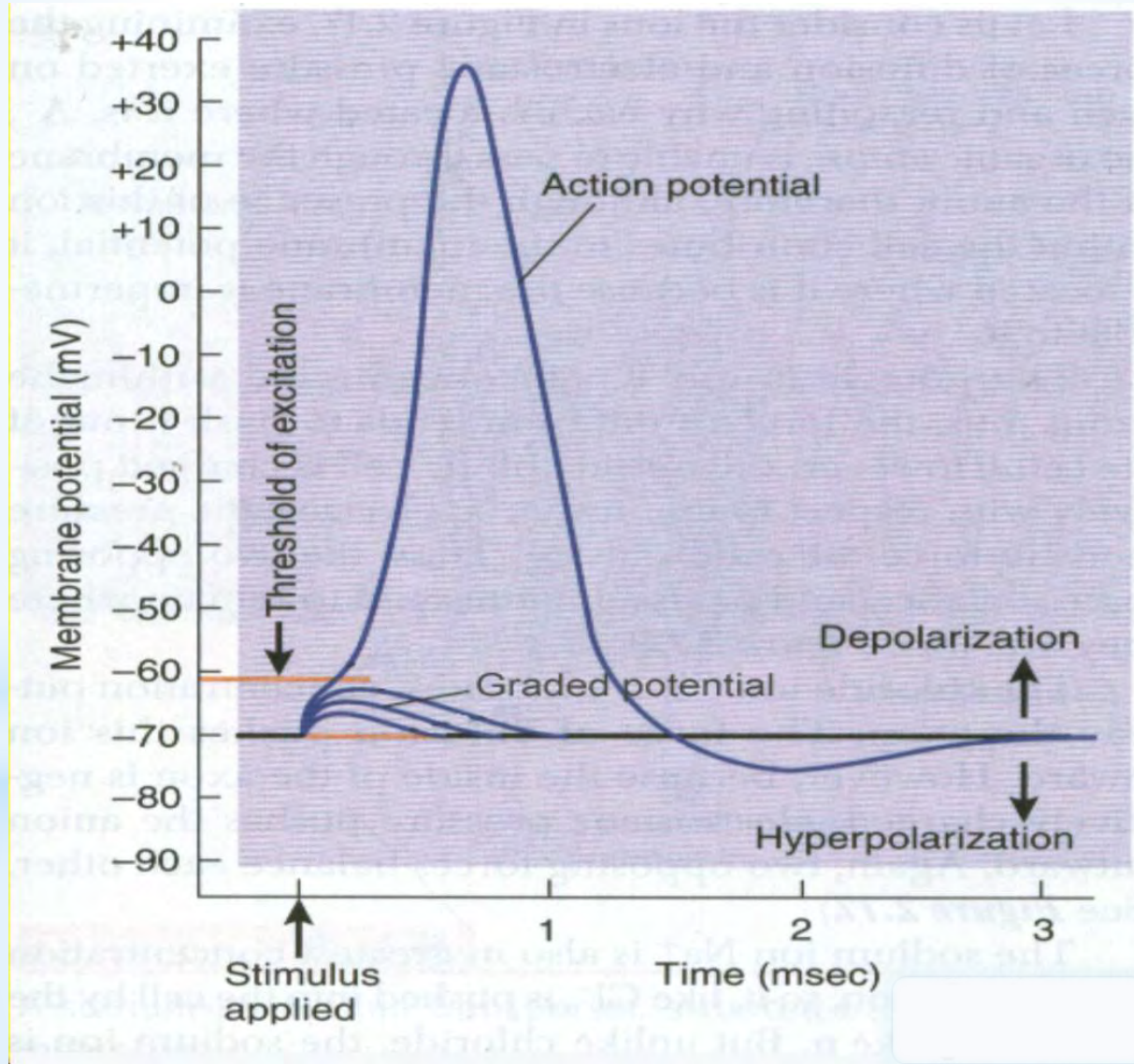


Electric signals trigger transmission of ions and neurotransmitters via the synapse to other cells.



The cell membrane contains channels through which specific ions flow. The membrane maintains a “concentration gradient”, which can be opposed by a growing electrical potential that opens channels and allow ions to flow into or out of the cell, depending on the type of channel.

An increase in cell membrane electrical potential is reflected graphically by a “spike”. This is also known as an “action potential.”



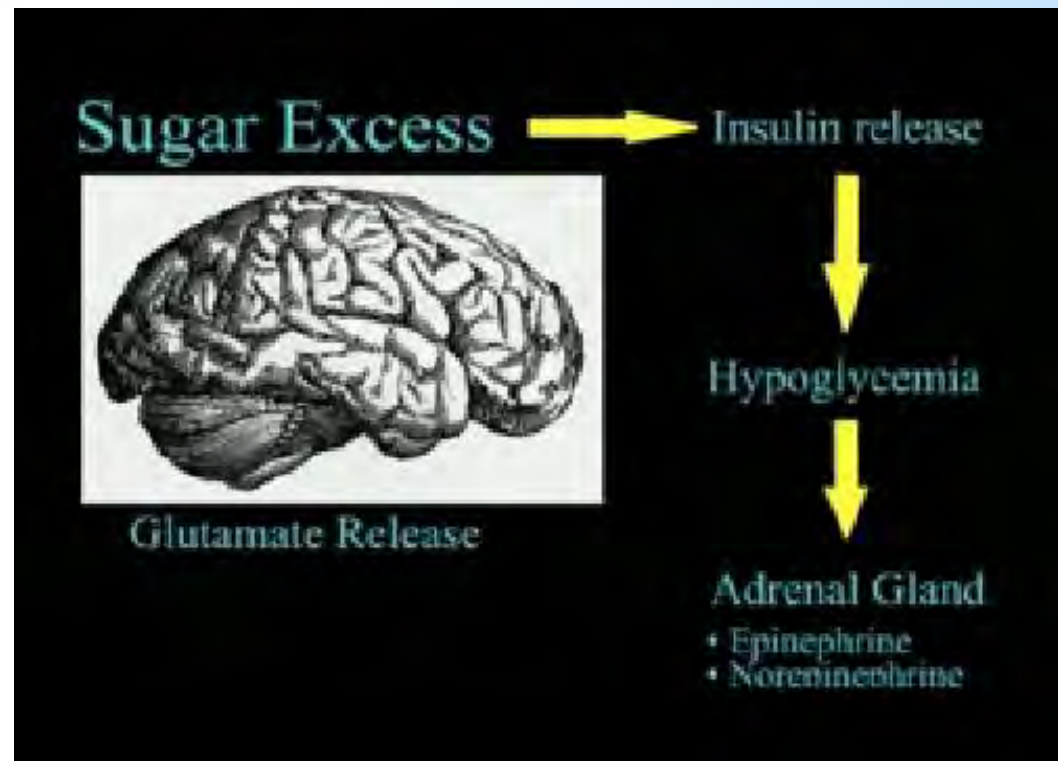
← “Resting State”

Video courtesy of Leslie Samuel of Interactive Biology

<http://www.interactive-biology.com/>

<http://interactivebiology.s3.amazonaws.com/mobile/IBTV008mb-shj.mp4>





“Bursting” of pancreatic β -cells causes release of insulin

Bursting Model Example with Synaptic Noise

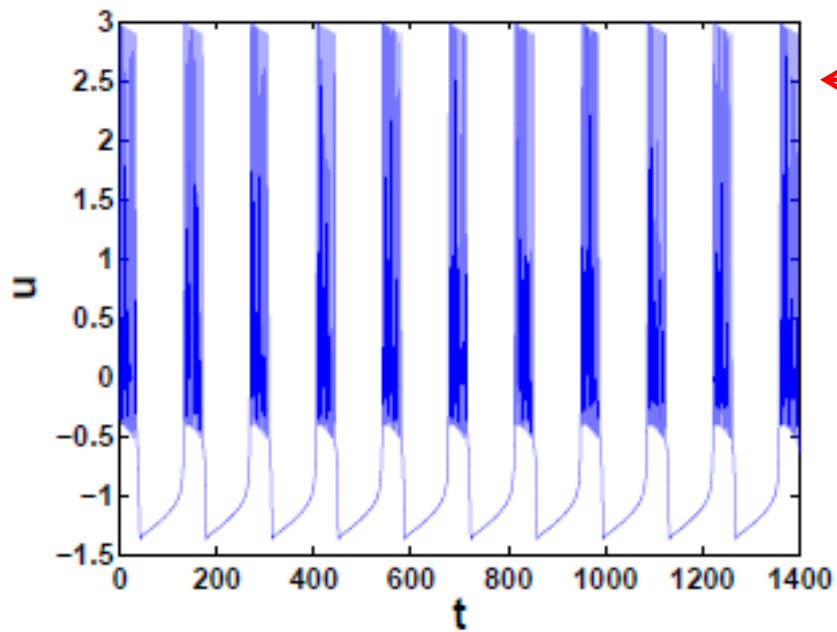
fast

$$(1) \frac{du}{dt} = y + \sigma \Gamma_t$$

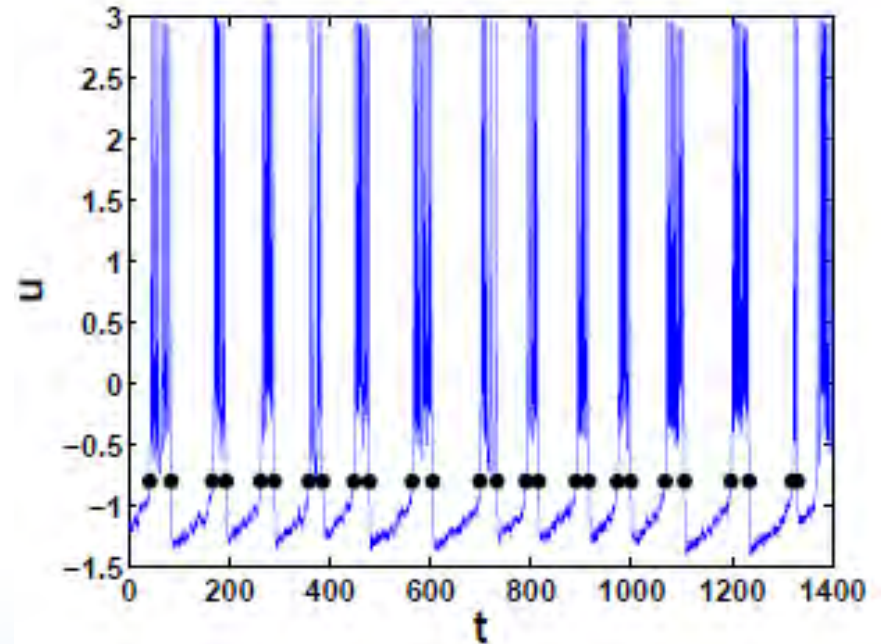
$$(2) \frac{dy}{dt} = -0.25((u - 1.6)^2 - 0.7^2)y - u^3 + 3(u + 1) - z - 0.0025[4(u + 0.954) - z]$$

$$(3) \frac{dz}{dt} = 0.0025[4(u + 0.954) - z] \quad \textit{slow}$$

- $\sigma \Gamma_t$ is a “white noise” term (formally: Gaussian term)
- y represents the influences of the various ionic currents

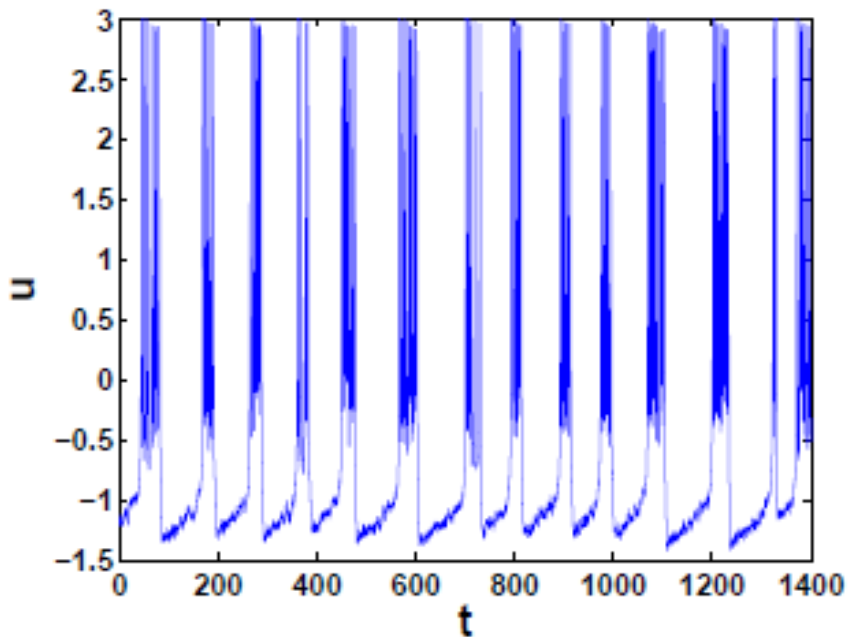


← Noise level $\sigma = 0$ (deterministic)

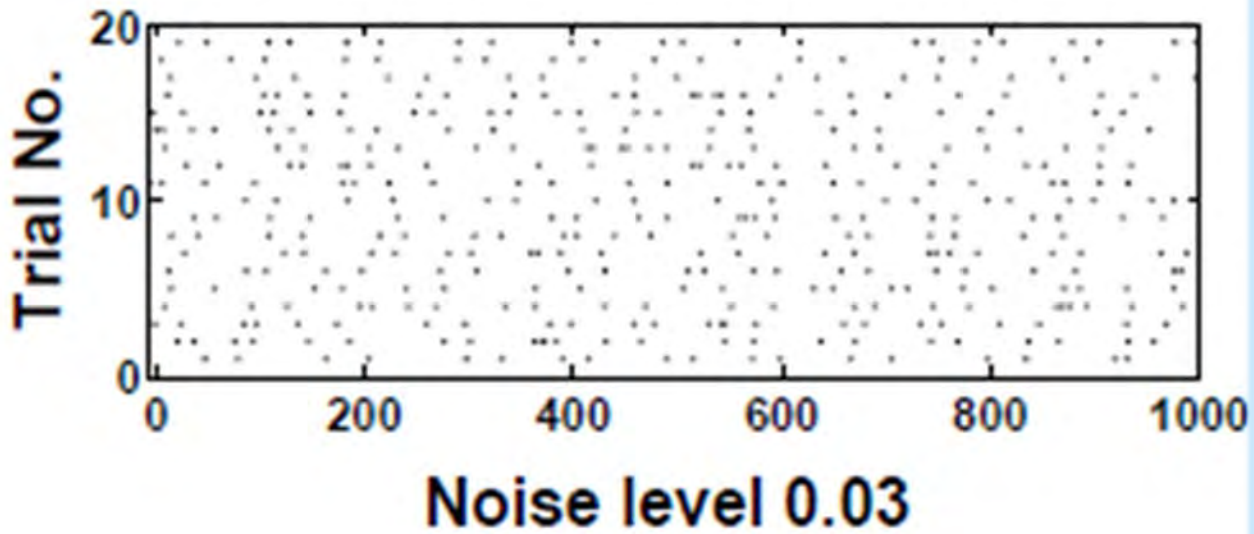
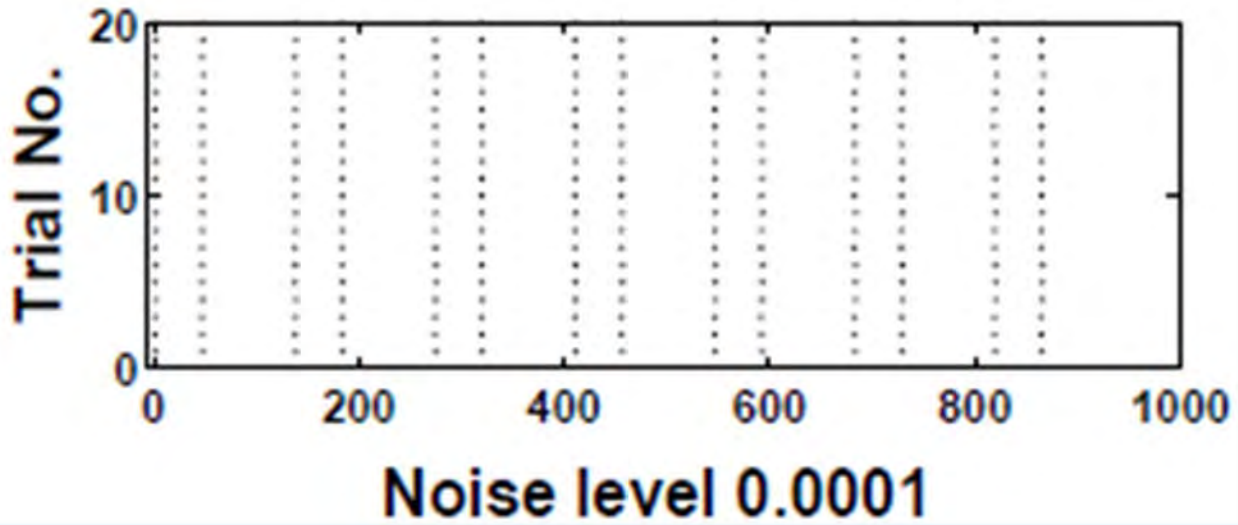


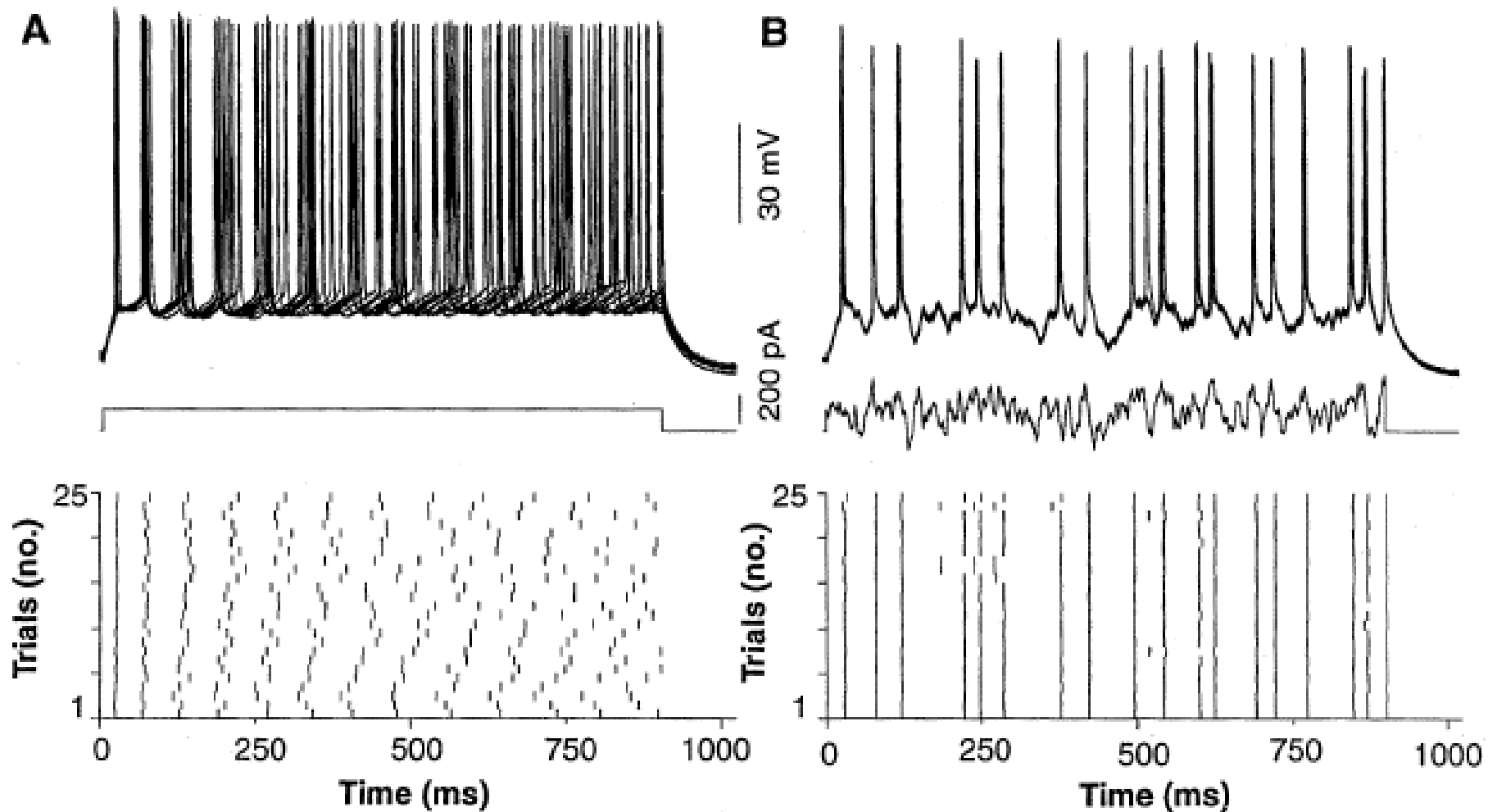
← Noise level $\sigma = 0.03$

Black dots represent beginning and ending of a "burst".

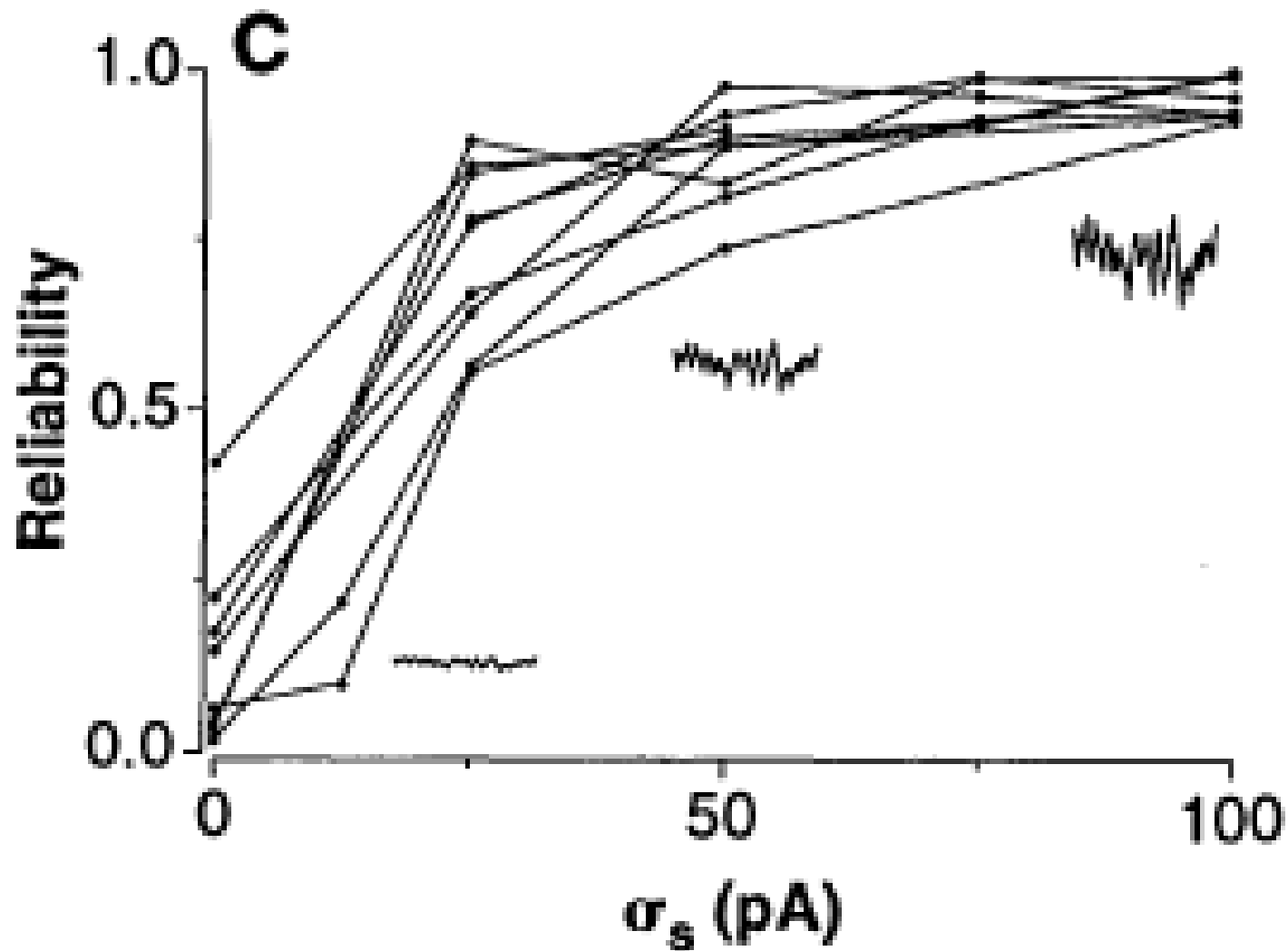


RASTER PLOTS





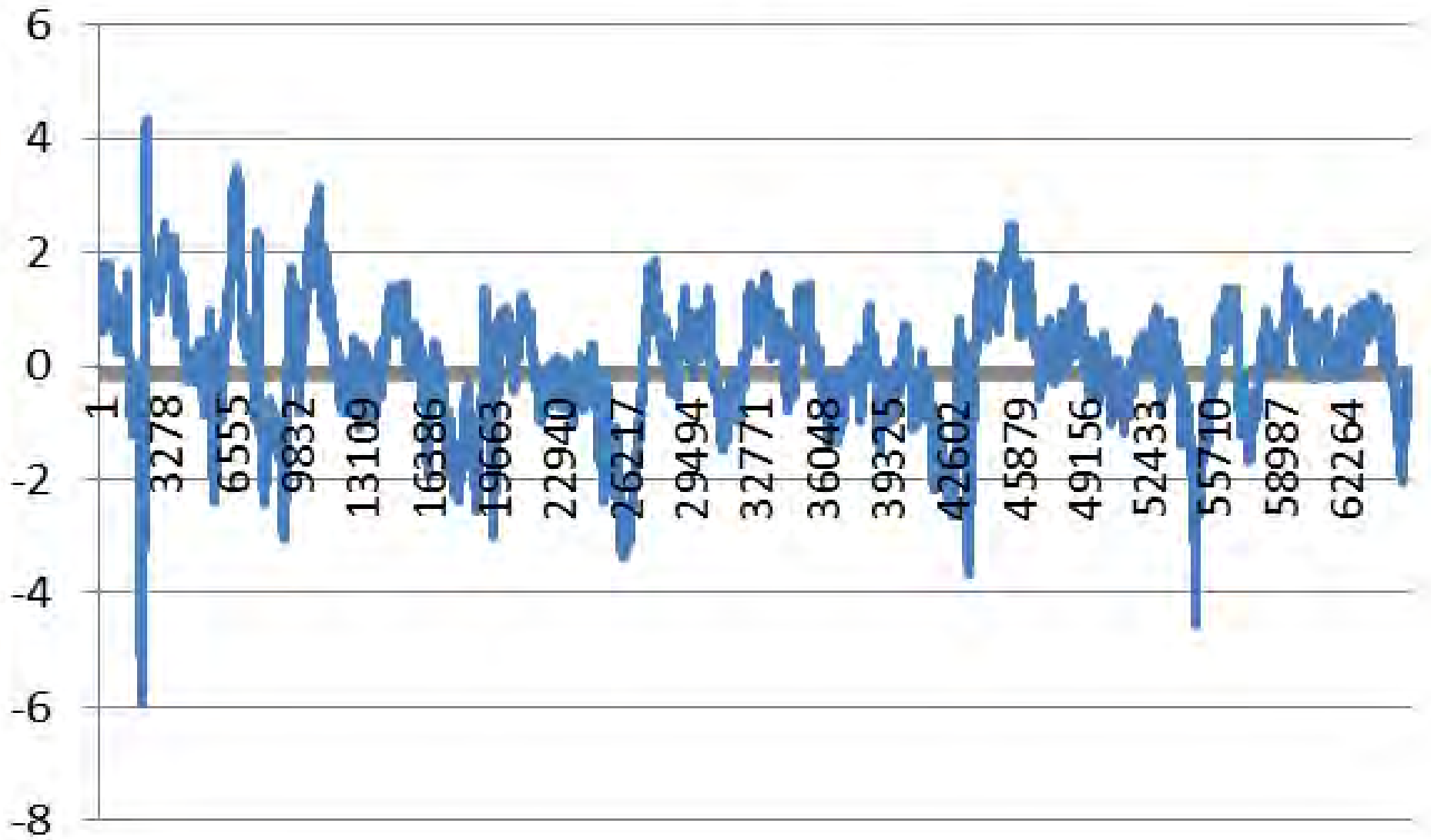
*** RELIABILITY OF SPIKE TIMING
IN NEOCORTICAL NEURONS**



* RELIABILITY INCREASES WITH NOISE!



* MAZE AS A CORTICAL STIMULUS



First define an "event"

Then try to reproduce the "event" . . . RELIABLY

INDIVIDUAL STATS of BREAKFAST TIMES

1. Convert times to decimal numbers (apply military time). Examples:
 - a. 6:15 AM = 6.25 hrs
 - b. 1:43 PM = 13.72 hrs (since $13 = 12 + 1$ and $43/60 \approx .72$)
2. Enter as a list in calculator: STAT > EDIT > 1:Edit, then enter values directly into L1.
3. Calculate stats: STAT > CALC > 1: 1-Var Stats, then press [2nd] [1] [ENTER]
 - a. Mean: _____
 - b. SD: _____
 - c. 5-number summary:
Min: _____ Q1: _____ Med: _____ Q3: _____ Max: _____
 - d. CV: _____ (SD/Mean) ~ *post on large class graph*
4. A raster plot is a graphical display of “tick marks” which correspond to the times at which action potentials of a neuron occur.
 - a. As an experiment or simulation is repeated multiple times, “tick marks” are recorded by vertically stacking them for each trial.
 - b. You will stack your breakfast times with each day being a “trial” on the given raster plot (*on back*). Use small sticky dots as tick marks.
 - c. Make your time axis (horizontal axis) so that all of your data will fit.
For example, if Min = 6.22 and Max = 8.73, then your axis should range from 6 to 9.

NAME _____

CODE NAME _____

INDIVIDUAL RASTER PLOT

DAY 21	_____
DAY 20	_____
DAY 19	_____
DAY 18	_____
DAY 17	_____
DAY 16	_____
DAY 15	_____
DAY 14	_____
DAY 13	_____
DAY 12	_____
DAY 11	_____
DAY 10	_____
DAY 9	_____
DAY 8	_____
DAY 7	_____
DAY 6	_____
DAY 5	_____
DAY 4	_____
DAY 3	_____
DAY 2	_____
DAY 1	_____

Time of Breakfast

NAMES _____

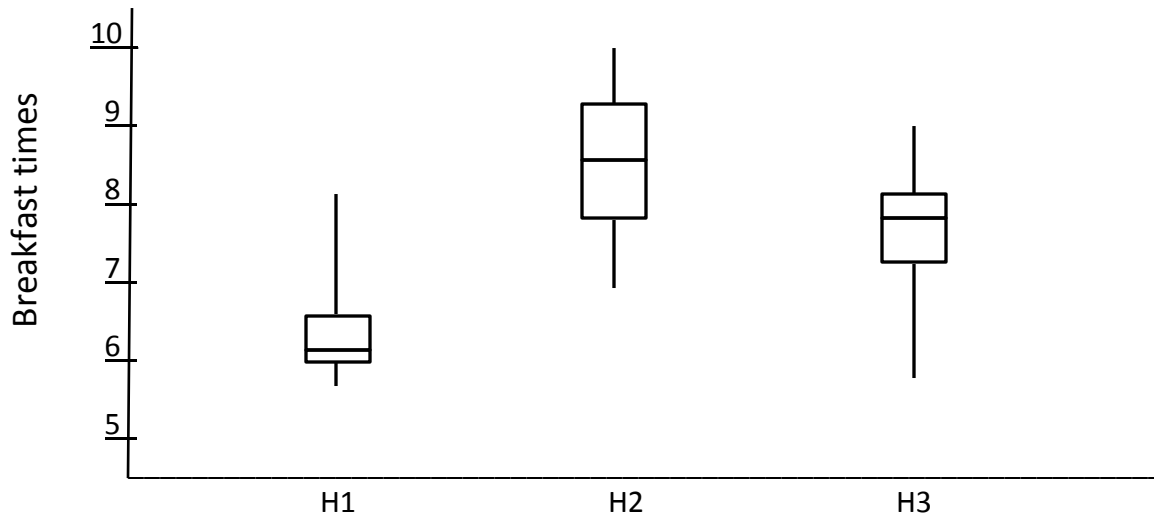
SAM HOUSTON GROUP _____

1. As a group, create a graph containing each of your five number summaries on the large graph paper provided to you. Determine minimum and maximum values of vertical axis.

Example

Suppose you have the following 5-number summaries:

H1:	5.75	6	6.03	6.55	8.11
H2:	6.93	7.77	8.5	9.35	9.99
H3:	5.89	7.37	7.9	8.2	8.97



2. Compare & Contrast

- a. Within your group, who has the most variable breakfast times?

- b. Whose data is most skewed? How can you tell?

3. Make a group raster plot on another large graph. Create your time axis so that all of the groups' values will fit. Use different colored sticky dots for each student to mark times.

Whose data is least predictable? most predictable?

1. Make a Bar-Time Histogram with your individual data.

a. Optimal bin width (bar width) selection process:

i. Let observation period $T =$ _____ minutes , and let $N =$ _____ bins.

Then each bin width $\Delta = T/N =$ _____

ii. Let $k_i =$ number of days your breakfast time is in bin i , (for $i = 1, \dots, N$).

$k_1 =$ _____ , $k_2 =$ _____ , etc.

iii. Calculate mean and variance of your $\{k_i\}$:

$$\bar{k} = \frac{1}{N} \sum_{i=1}^N k_i \quad \text{and} \quad var = \frac{1}{N} \sum_{i=1}^N (k_i - \bar{k})^2$$

iv. Calculate “cost function”:

$$C(\Delta) = \frac{2\bar{k} - var}{(21 \cdot \Delta)^2}$$

v. Repeat steps i through iv with $N = 10$ and $N = 20$.

Optimal bin width Δ is that which yields the **lowest** $C(\Delta)$.

b. Once you have determined your optimal bin width Δ , construct a relative frequency histogram.

Example: If you have 5 breakfast times in the third bin, then the relative frequency of the third bin is $5/21 \approx 0.24$. Therefore the height of bin #3 should be 0.24 on the histogram.

2. Using your optimal bin width Δ , which bins have more than \bar{k} breakfast times.

How many breakfast times are in these bins altogether? $b^* =$ _____

Your overall reliability, $R = \frac{b^*}{21} \approx$ _____

Use a large sticky dot to plot your reliability on the large class graph.

NAME _____

Bar-PSTH

BREAKFAST TIME HISTOGRAM OF INDIVIDUAL STUDENT FREQUENCIES

Relative Frequency



BREAKFAST TIMES